Quantum Gravity and Black Holes

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March 30, 2007

Outline

Classical setting

Quantum theory

Gravitational collapse in quantum gravity

Summary/Outlook

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- It appears in the kinetic term (and propagator) of any theory:

$$S = \int dt q_{ab} \dot{x}^a \dot{x}^b$$

q^{ab} is the (usually invisible) Euclidean 3-metric in particle mechanics, and in

$$S = \int d^4x \sqrt{-g} g^{ab} \partial_a \phi \partial_b \phi$$

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  g^{ab} is usually the Minkowski metric.
- A fixed g_{ab} and its symmetries (eg. Poincare invariance) are an essential part of standard quantization methods (– there is no Schrodinger eqn, QED, QCD, even string theory, without a fixed metric).

General Relativity I

- General relativity and its extensions are theories with a dynamical metric.

$$G_{ab}(g) = 8\pi T_{ab}(\phi, g)$$

- There is no fixed background metric: metric and matter are solutions of coupled equations.

There are only two types of classical theories: background dependent and independent.

General Relativity II

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- Spherically symmetric vacuum solution: Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

  Stable and singular solutions that arise as the end point of gravitational collapse.
  Have an event horizon at $r = 2M$: for $r < 2M$ light cones tip toward the singularity.

- Homogeneous and isotropic cosmology (FRW)

  $$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

  with a fluid matter source with energy density $\rho(t)$, pressure $P(t)$, and equation of state $P = k\rho$.

- Gravitational waves: one writes

  $$g_{ab} = \eta_{ab} + h_{ab}$$

  with η_{ab} treated as a fixed background. h_{ab} satisfies the wave equation to linear order.

For special solutions like these there are preferred frames: the coordinate transformations that leave the metric form invariant.

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- In quantum field theory it is the Minkowski metric.

Quantization II: Semiclassical gravity

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- How is $|\psi\rangle$ chosen? What is g if it is a linear combination of squeezed? etc.
- Is it consistent? The r.h.s might require regularization and renormalization.

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- Hawking radiation: Schwarzschild metric
- Structure formation in cosmology: FRW metric

The cosmological constant problem arises in this approximation: if $|\psi\rangle$ is a vacuum state $\langle \hat{T}_{ab} \rangle$ is ultraviolet divergent.

A metaphor

To see how odd this approximation can be consider this “inverse” problem. Pick a quantum state $\psi(\vec{x})$ and an energy E,

and “solve” the Schrodinger equation

$$H\psi = -\hbar^2 q^{ab}\partial_a\partial_b\psi = E\psi$$

for possible metrics q_{ab}.

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NO SUCH THEORY IS KNOWN

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The rest of the talk is concerned with the first approach.

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This suggests how to quantize a particle without a background: associate a one dimensional Hilbert space at each point.

A particle as a matrix model

Lattice $x_n = n\lambda$, $(n = \cdots - 1, 01, \cdots)$.

- Hilbert space $|k\rangle$
- $\hat{x}|k\rangle = \lambda k|k\rangle$
- $\widehat{e^{i\lambda p}}|k\rangle >= |k+1\rangle$ (hopping operator)

This is a scale (λ) dependent quantization of the particle. Kinetic energy operator is written using $\hat{T}_\lambda = \widehat{e^{i\lambda p}}$.

Hydrogen atom: Is there a $1/x$ operator?

$$\frac{1}{|x|} = \left(2\frac{1}{i\lambda}T^*\{T, \sqrt{|x|}\}\right)^2$$

Thiemann (1996)

- Spectrum of $1/x$ operator is bounded.
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- Spectrum of $1/x$ operator is bounded.
- Hydrogen atom spectrum is reproduced for $\lambda << $ Bohr radius
- Trick used in cosmology to get bounded inverse scale factor (Bojowald (2001))
- Can be used to get a quantum gravity corrected wave equation: replace scale factor by its expectation value in some state.

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The problem

Find the quantum theory of the gravity + scalar field in spherical symmetry: Fields $g_{ab}(r,t)$ and $\phi(r,t)$

Metric:

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- Put system in Hamiltonian form (ADM variables for GR)
- Fix a time gauge condition

Resulting theory has two fields and their conjugate momenta. There is a constraint due to residual gauge symmetry (like Gauss law in EM). Evolution via Hamilton's equation

(VH, O. Winkler Phys.Rev. D71 (2005) 104001)

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- Define action of basic operators
- Construct Hamiltonian and constraint operators
- Operators for trapped surface detection

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What is a black hole in quantum gravity?

- The event horizon is a global classical entity: “The boundary of the past of future null infinity”
- Not useful for local physics even classically: How would you determine if you walk into a black hole? Need a local test.

Trapped surfaces I

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- *S* is not trapped : $\theta_-^S < 0$ and $\theta_+^S > 0$.
- *S* is trapped: $\theta_+^S < 0$ and $\theta_+^S < 0$.
- Space can be divided up into domains containing trapped and untrapped surfaces.

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- $\theta_\pm(g, \phi, P_g, P_\phi)$ is a phase space variable
- Construct the corresponding operator in a background independent quantization.
- A state is a quantum black hole if for all r

$$\langle \hat{\theta}_-(r, t) \rangle < 0,$$

and

$$\langle \hat{\theta}_+(r, t) \rangle < 0$$

for $r < r^*$ for some r^*.

(VH, O. Winkler Class.Quant.Grav. 22 (2005) L135)

Gravitational collapse in quantum gravity

A complete regulated theory is ready for calculation. Being implemented numerically.

- Pick initial state
- Evolve with Hamiltonian: singularity free (VH, O. Winkler, Class. Quant. Grav. 22 (2005) L127)
- Test state for trapping at each time step
- Ensure constraint satisfied at each time step

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- How does the semiclassical approximation arise?