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Nonlinear Interactions in Dense Quantum Plasmas

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OUTLINE

Introduction

- The field of quantum plasmas is gaining momentum.
- Quantum plasmas are ubiquitous in ultrasmall electronic devices and micromechanical systems, in intense laser–solid density plasma interaction experiments and in microplasmas, and in superdense astrophysical objects (neutron stars and white dwarfs).
- Traditionally, quantum effects are important when the de Broglie wavelength of the charge carriers (electrons, holes/positrons) is comparable to the dimension of the system.
- Here quantum mechanical effects (e.g. tunneling) can play an important role at nanoscales in dense plasmas.

Introduction (Continued)

- We are thus dealing with extremely high density and low-temperature plasmas, in contrast to traditional plasmas that have high temperature and low density.

- In dense quantum plasmas, there are quantum forces due to the strong electron/positron (hole) density correlations (the Bohm potential), and the Bohr magnetization caused by the electron/positron-$1/2$ spin effect. The quantum statistical description for a Fermi plasma yields a new pressure law owing to the Fermi-Dirac statistics.

- At quantum scales, we will have new wave modes, new aspects of quantum plasmonic fluid turbulence, and nonlinear structures (dark & gray solitons, quantum vortices, trapped photons in quantum electron holes) in quantum plasmas.

Properties of Dense Quantum Plasmas

- Quantum effects can be measured by the thermal de Broglie wavelength of the particles composing the plasma

$$\lambda_B = \frac{\hbar}{mV_T},$$

which roughly represents the spatial extension of a particle's wave function due to quantum uncertainty. For classical regimes, the de Broglie wavelength is so small that particles can be considered as pointlike, and therefore there is no overlapping of the wave functions and no quantum interference.

Properties of Dense Quantum Plasmas (Continued)

- It is reasonable to postulate that quantum effects start playing a significant role when the de Broglie wavelength is similar to or larger than the average interparticle distance $n^{-1/3}$, i.e. when $n\lambda_B^3 \geq 1$.

- Quantum effects become important when the temperature is lower than the so-called Fermi temperature T_F, defined as

$$T_F = \frac{1}{2} m V_F^2 \equiv E_F = \frac{\hbar^2}{2m}(3\pi^2)^{2/3} n^{2/3}.$$

- We note that when the temperature approaches T_F, the relevant distribution changes from the Maxwell-Boltzmann to the Fermi-Dirac.

Properties of Dense Quantum Plasmas (Continued)

- The relevant velocity for a Fermi-Dirac distribution is

$$V_F = (2E_F/m)^{1/2} = \frac{\hbar}{m}(3\pi^2 n)^{1/3}.$$

- It is easy to see that

$$\chi = \frac{T_F}{T} = \frac{1}{2}(3\pi^2)^{2/3}(n\lambda_B^3)^{2/3}.$$

Thus quantum effects become important when $\chi \geq 1$.

Properties of Dense Quantum Plasmas (Continued)

- The Fermi screening scalelength

$$\lambda_F = \frac{V_F}{\omega_p}$$

is the quantum analogue of the Debye radius.

- The quantum coupling parameter

$$G_q = \frac{E_{int}}{E_F} \sim \left(\frac{1}{n\lambda_F^3}\right)^{2/3} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2,$$

is completely analogous to the classical one when one substitutes $\lambda_F \to \lambda_D$.

Model Equations

- The most fundamental model for the quantum N body problem is the Schroedinger equation for the N-particle wave function $\psi(x_1, x_2, x_N, t)$. We assume that the N-body wave function can be factored into the product of N-one body functions:

$$\psi(x_1, x_2, ...x_N, t) = \psi_1(x_1, t)\psi_2(x_2, t)...\psi_N(x_N, t)$$

  For Fermions a weak form of the Pauli exclusion principle is satisfied if none of the wave functions on the right-hand side are identical.

- We can then introduce the density matrix formalism, i.e. use the Wigner and Hartee models.

Model Equations (Continued)

- The Wigner-Poisson model for our purposes is then

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \frac{e}{m} \nabla \phi \cdot \nabla_{\mathbf{v}} f \approx \frac{\hbar^2}{24 m^3} \nabla \nabla^2 \phi \nabla_{\mathbf{v}}^3 f$$

$$\nabla^2 \phi = 4\pi e \left(\int f d^3 v - n_0 \right).$$

- N.B.: The right-hand side of the Wigner equation is due to the strong electron density correlation. It has been obtained by using a small expansion coupling parameter.

Quantum Hydrodynamical (QHD) Model

- We take the moments of the Wigner equation and obtain for the quantum-electron fluid

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$m\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = e\nabla\phi - \frac{1}{n}\nabla P + \mathbf{F}_Q,$$

where ϕ is determined from $\nabla^2\phi = 4\pi e(n - n_0)$, and for the FD plasma we have

$$P = \frac{mV_F^2}{3n_0^2}n^3 \quad \text{and} \quad \mathbf{F}_Q = \frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) \equiv -\nabla\phi_B.$$

Quantum Hydrodynamical Model (Continued)

Introduce the effective wave function

$$\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\,\exp(iS(\mathbf{r},t)/\hbar)$$

where S is defined according to $m\mathbf{u} = \nabla S$ and $n = |\psi|^2$. It is easy to show that the QHD equations are equivalent to

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi + e\phi\psi - \frac{mV_F^2}{2n_0^2}|\psi|^{4/D}\psi = 0$$

and

$$\nabla^2\phi = \frac{e}{\epsilon_0}(|\psi^2| - n_0)$$

- Without the ϕ term, the NLSE is similar to that found in BECs. For $D = 1$, we have analytical solutions (PRL **85**, 1146, 2000).

Aspects of Turbulence and Structures at Quantum Scale

Normalized system of equations for plasmons

$$i\frac{\partial\Psi}{\partial t} + A\nabla^2\Psi + \varphi\Psi - |\Psi|^{4/D}\Psi = 0,$$

$$\nabla^2\varphi = |\Psi|^2 - 1,$$

where A represents the quantum coupling strength. Conserved quantities:

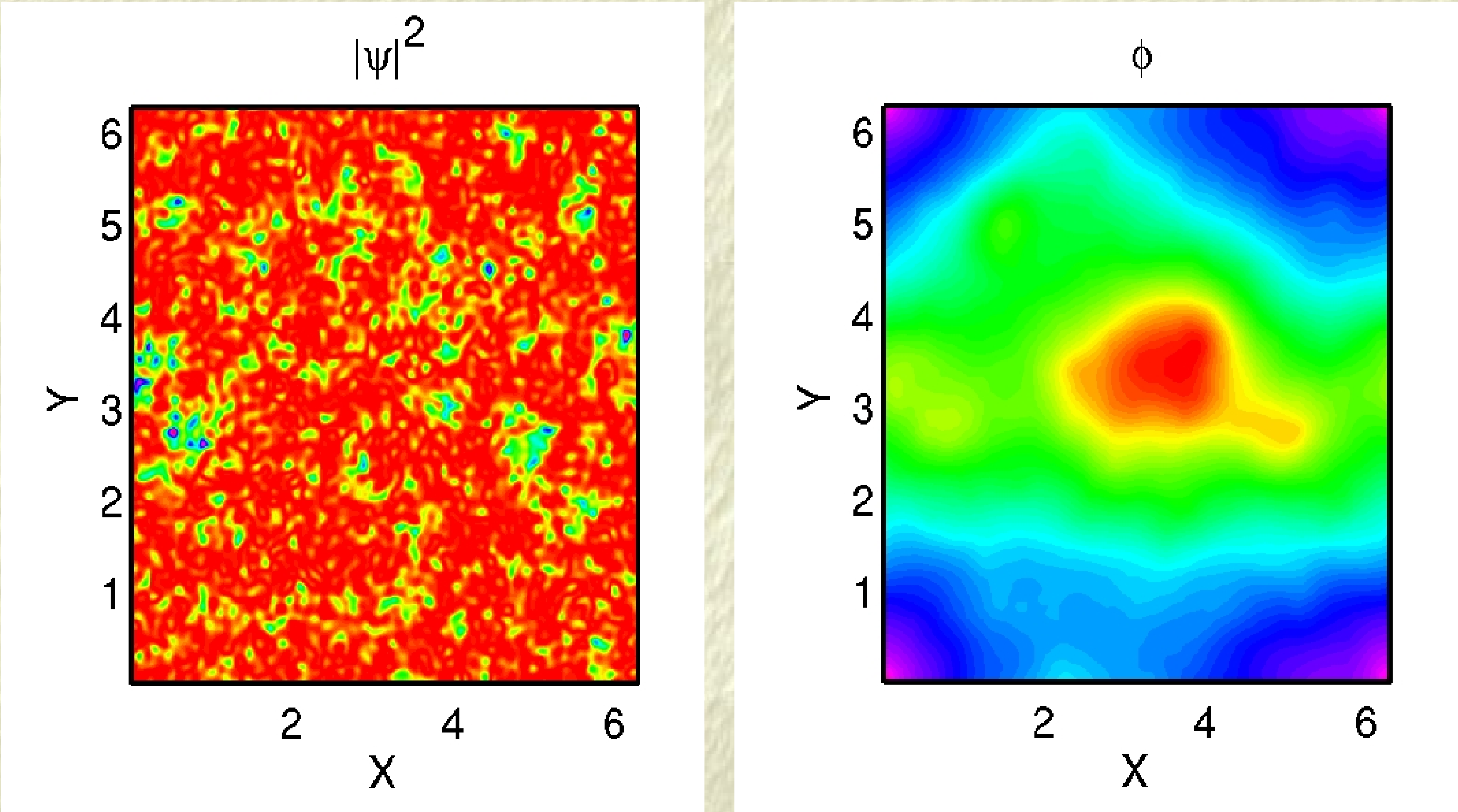
$$N = \int |\Psi|^2 \, d^3x$$

$$\mathbf{P} = -i\int \Psi^*\nabla\Psi \, d^3x$$

$$\mathbf{L} = -i\int \Psi^*\mathbf{r}\times\nabla\Psi \, d^3x$$

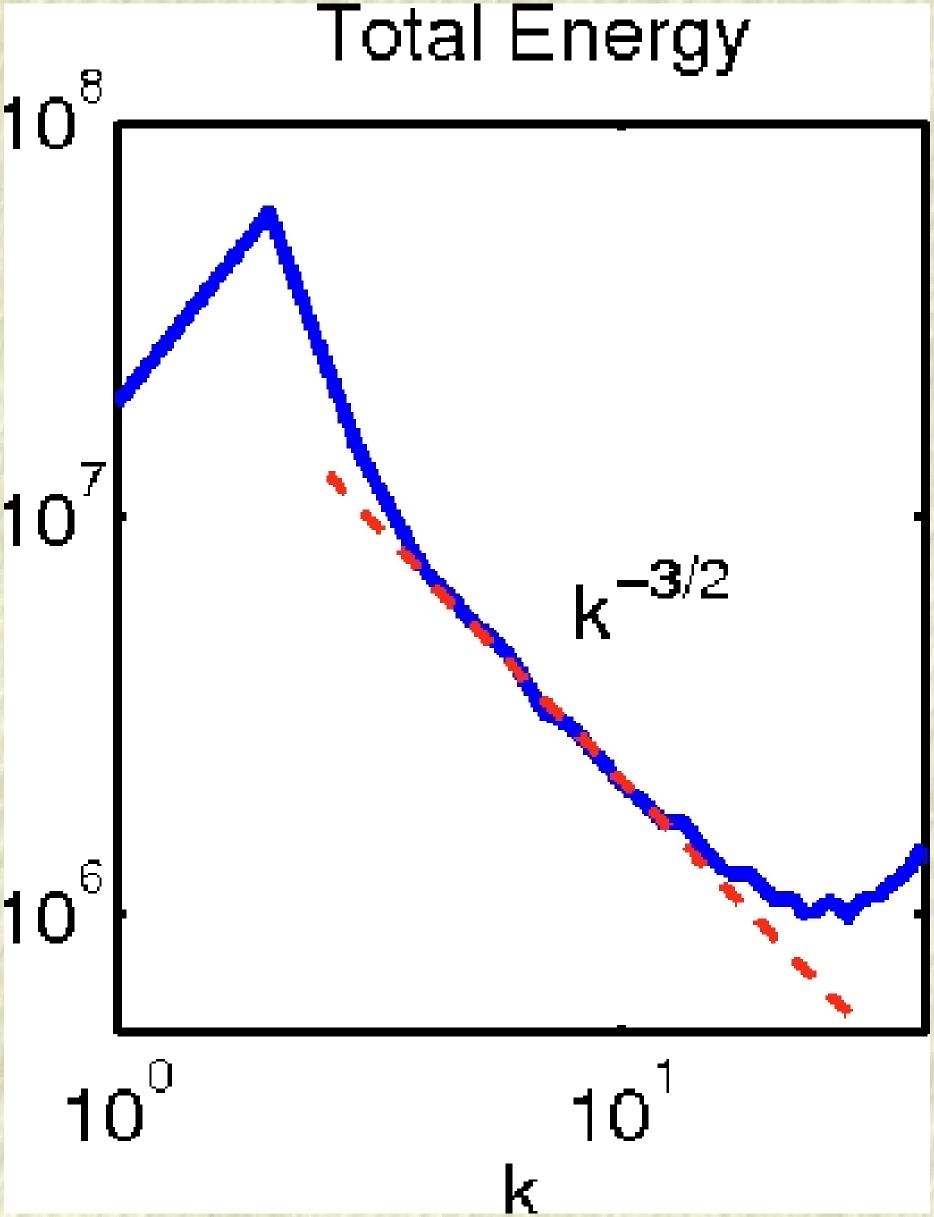
$$\mathcal{E} = \int \left[-\Psi^* A\nabla^2\Psi + |\nabla\varphi|^2/2 + |\Psi|^{2+4/D} D/(2+D)\right] d^3x$$

Dual Cascade (movies)

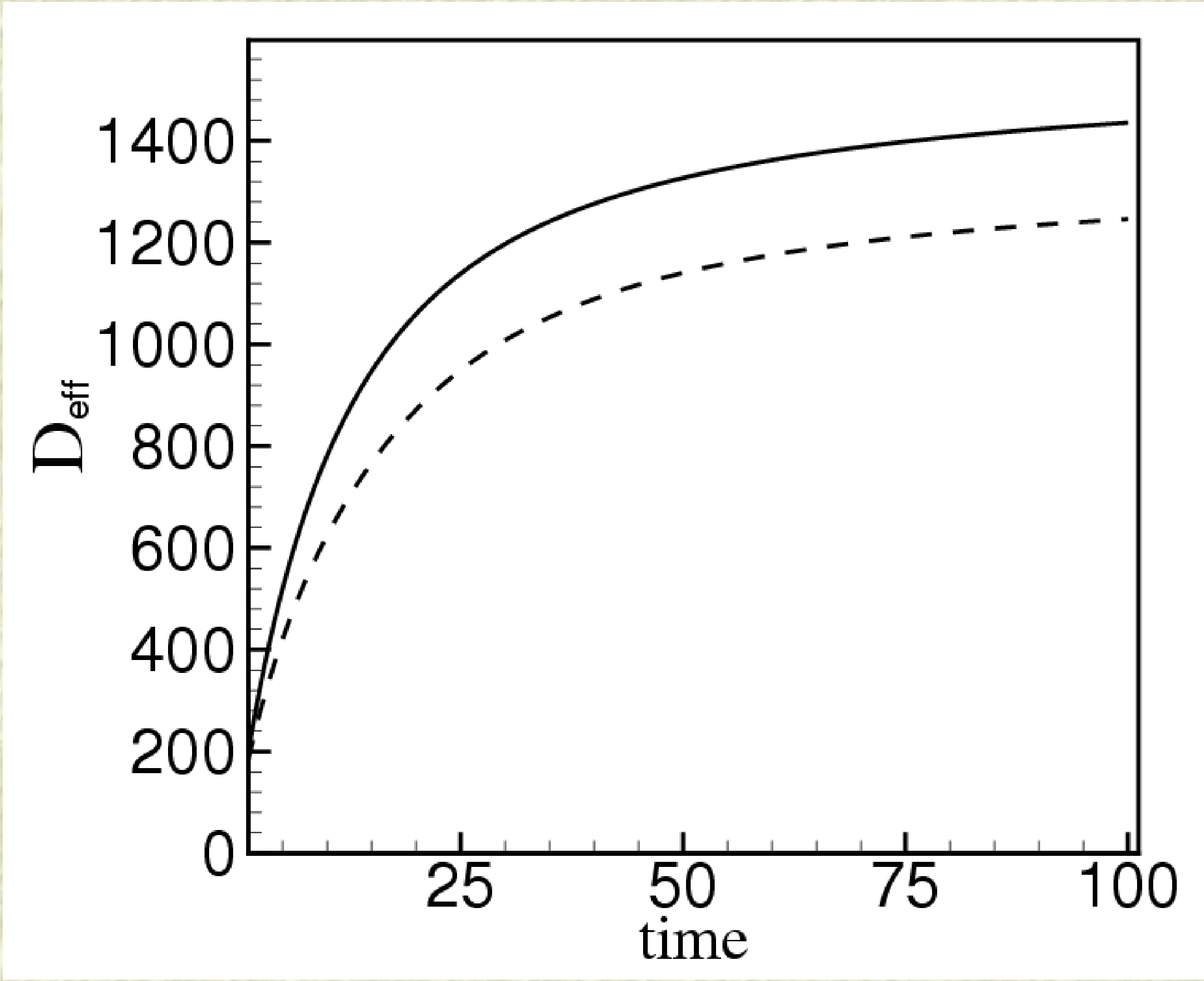


Dastgeer & Shukla, PRL (submitted)

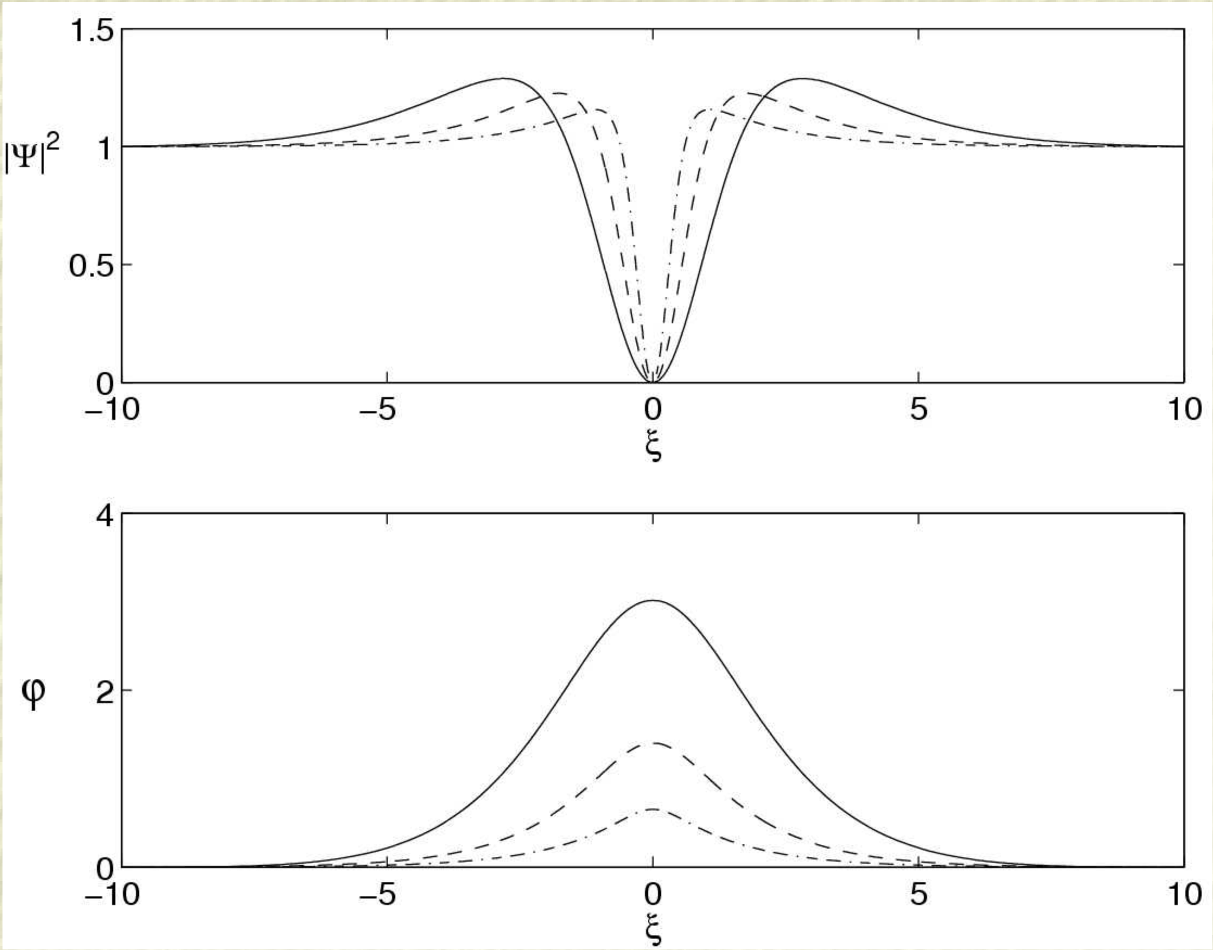
Iroshnik-Kraichnan Spectrum For Quantum Fluidics



Turbulent Transport – Effective Diffusion Coefficient



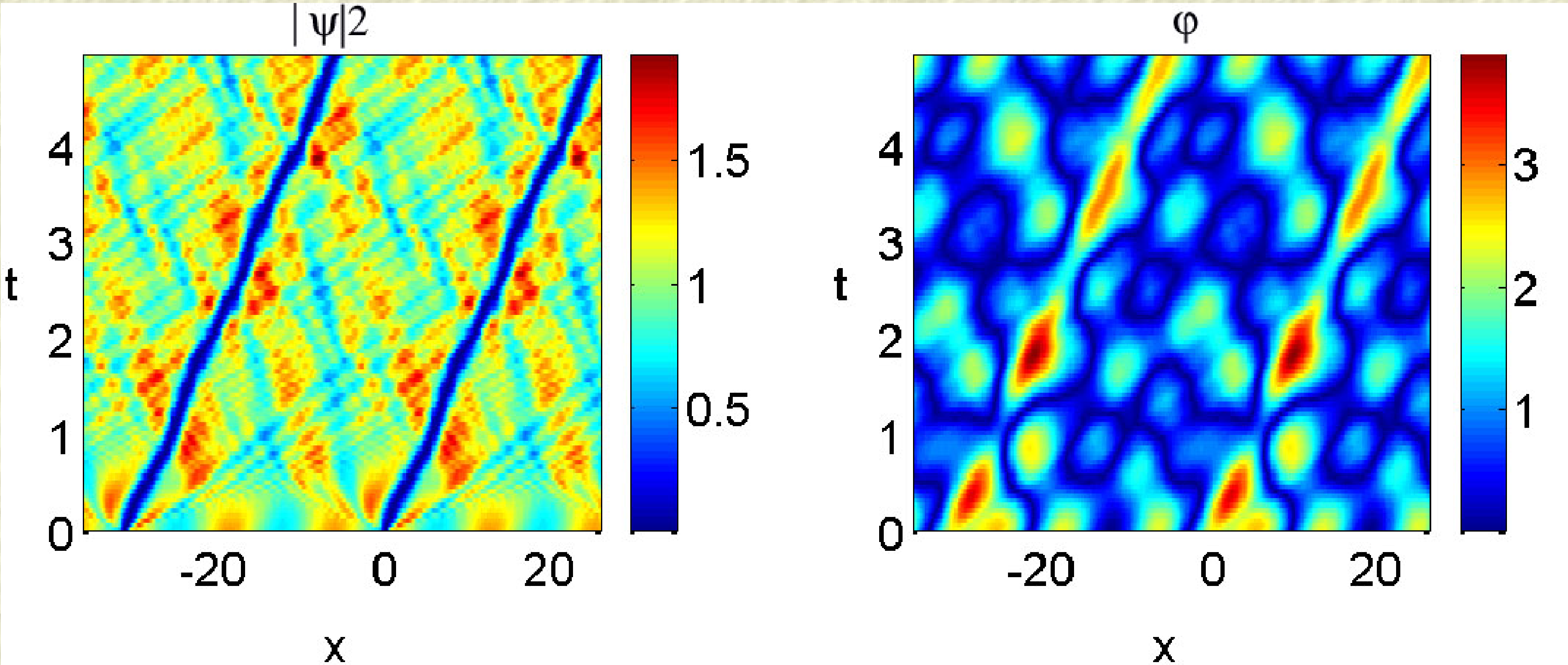
1D Quantum Electron Hole (Dark Soliton)



Shukla & Eliasson, PRL **96**, 245001 (2006)

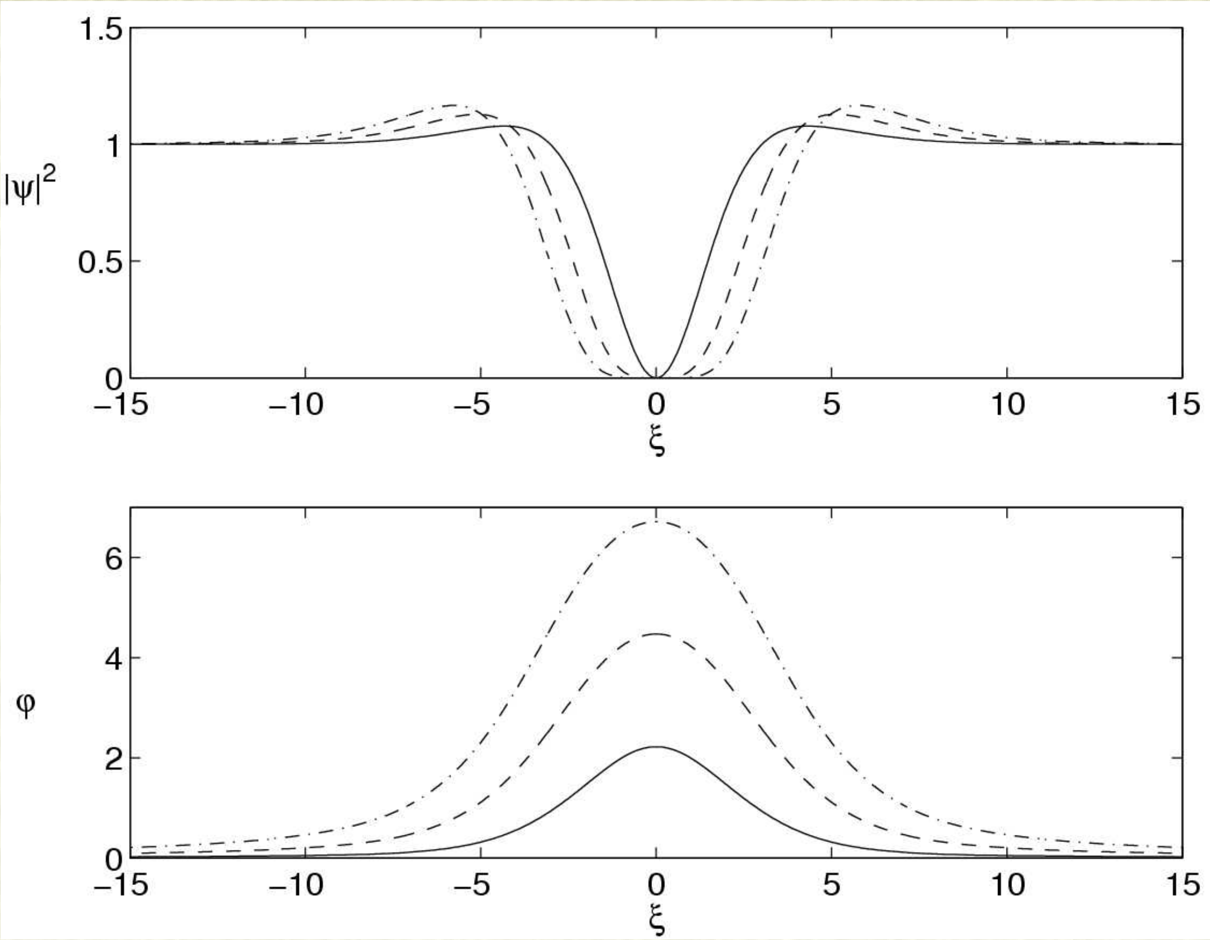
Dynamics of Quantum Electron Holes

Electron density (left) & electrostatic potential (right)



Shukla & Eliasson, PRL **96**, 245001 (2006)

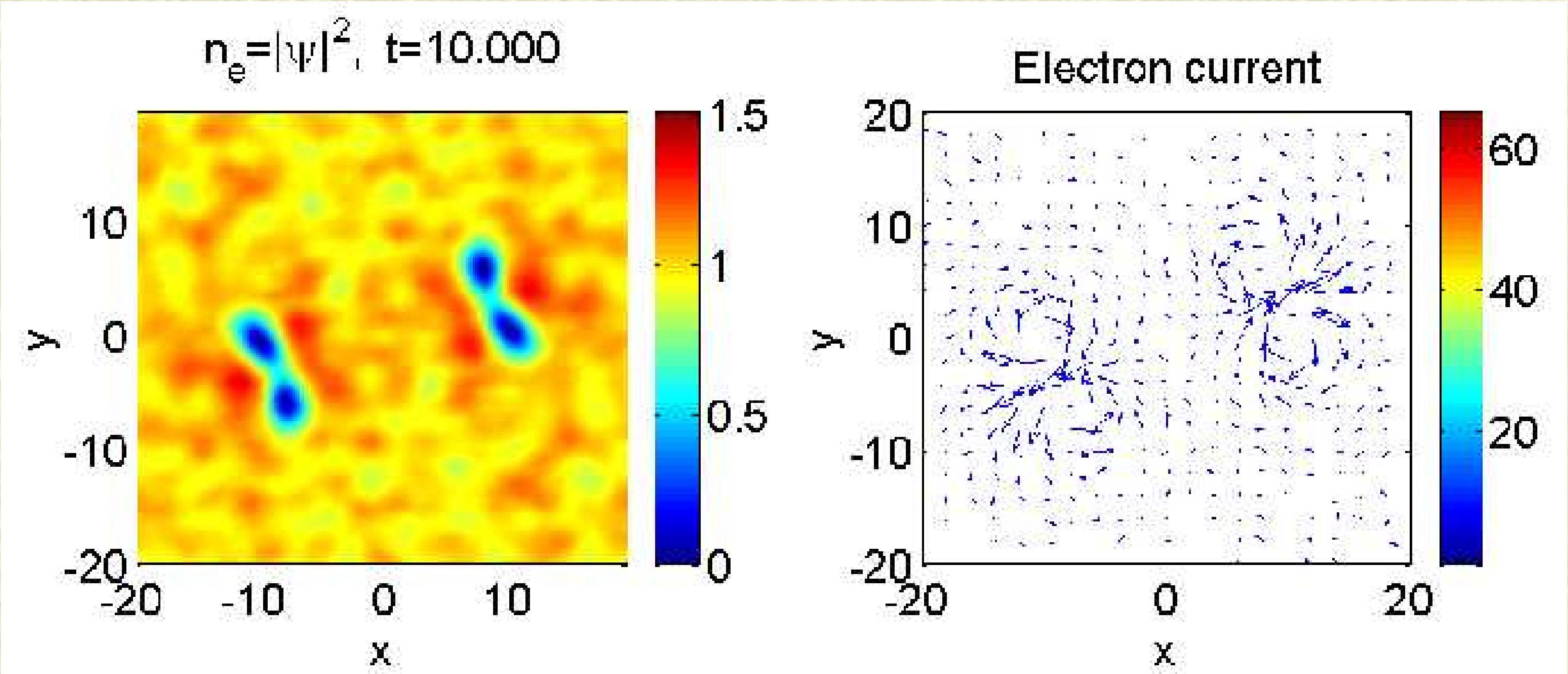
2D Quantum Electron Vortices



Shukla & Eliasson, PRL **96**, 245001 (2006)

Interacting 2D Quantum Vortices

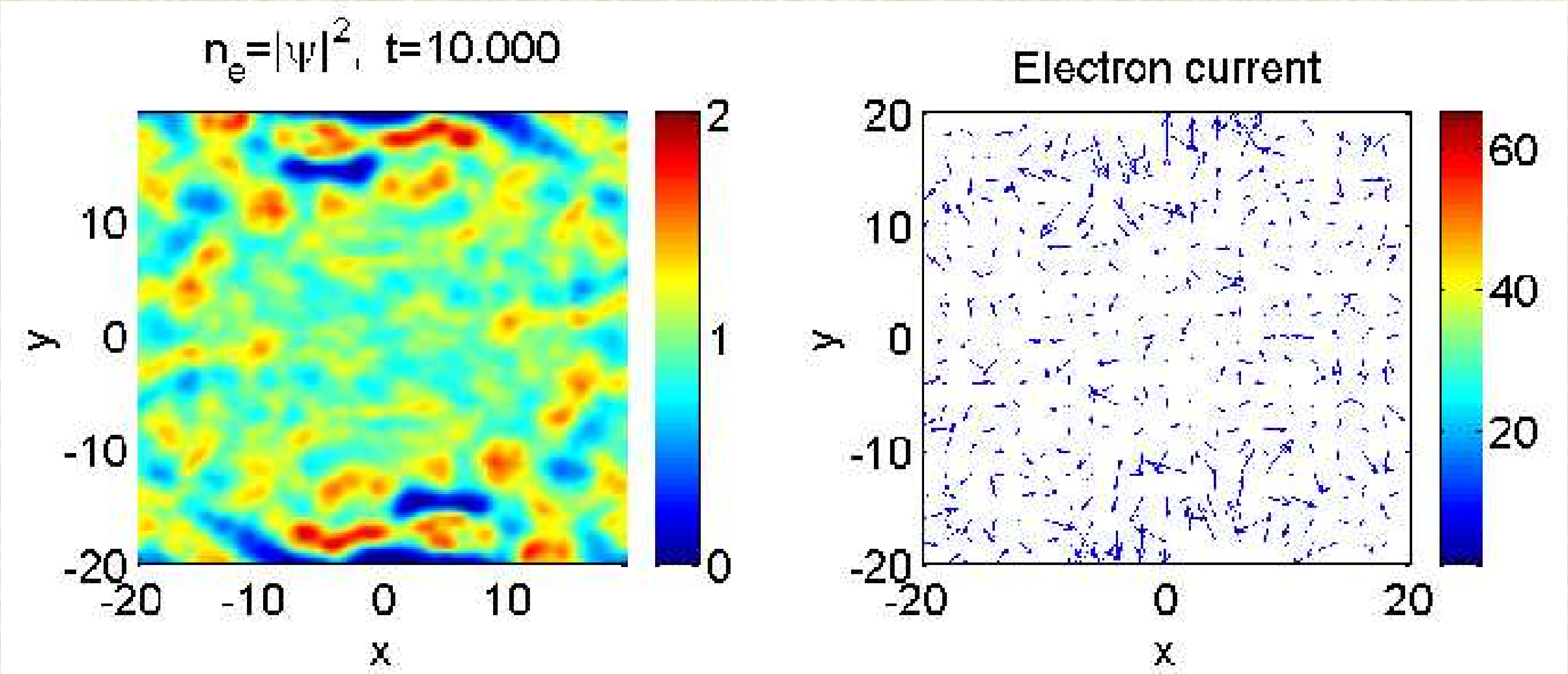
Single charge states ($n = 1$)



Shukla & Eliasson, PRL **96**, 245001 (2006)

Interacting 2D Quantum Vortices

Double charge states ($n = 2$)



Shukla & Eliasson, PRL **96**, 245001 (2006)

Electrostatic Wave Modes

- The plasmonic wave dispersion relation is

$$\omega^2 = \omega_p^2 + k^2 V_F^2 + \frac{\hbar^2 k^4}{4m^2}$$

- Quantum ion-acoustic waves

$$\omega^2 = \frac{k^2 C_s^2 (1 + H_F^2 k^2 \lambda_{DF}^2/4)}{1 + k^2 \lambda_{DF}^2 (1 + H_F^2 k^2 \lambda_{DF}^2/4)}$$

where

$$V_F = \left(\frac{T_{Fe}}{m_e}\right)^{1/2} \text{ and } C_s = \left(\frac{T_{Fe}}{m_i}\right)^{1/2}$$

$$H_F = \frac{\hbar \omega_p}{T_{Fe}}$$

Quantum Wave Modes (Continued)

- In an ultracold dense quantum plasma, we have

$$\omega = \left(\omega_p^2 + \hbar^2 k^4/4m^2\right)^{1/2}$$

for the plasmons, and

$$\omega = \frac{\hbar k^2}{2\sqrt{m_e m_i}}$$

for the quantum ion oscillations.

N.B.: The electron plasma frequency ω_p is extremely high in a super-dense plasma: $n_0 \sim 10^{25}\,\mathrm{cm}^{-3}$, $\omega_p \sim 10^{17}$ rad/s.

Excitation of ES Quantum Modes by Photons

- Both plasmons and quantum ion oscillations couple with photons nonlinearly. The governing equations are

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_p^2\right)\mathbf{A} + \omega_p^2 \frac{n_{e1}}{n_0}\mathbf{A} = 0$$

for the photons

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 + \frac{\hbar^2}{4m_e^2}\nabla^4\right) n_{e1} = \frac{n_0 e^2}{2m_e^2 c^2}\nabla^2|\mathbf{A}|^2$$

for the driven plasmons, and

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\hbar^2}{4m_e m_i}\nabla^4\right) n_{e1} = \frac{n_0 e^2}{2m_e m_i c^2}\nabla^2|\mathbf{A}|^2$$

for the driven quantum ion oscillations in an ultracold plasma.

Parametric Instability Growth Rates

- The SRS and SBS growth rates are

$$\gamma_R = \frac{\omega_p e K |\mathbf{A}_0|}{2\sqrt{2}\sqrt{\omega_0 \Omega_R} m_e c}$$

$$\gamma_B = \frac{\omega_p e K |\mathbf{A}_0|}{2\sqrt{2}\sqrt{\omega_0 \Omega_B} m_e m_i c}$$

Shukla & Stenflo, PoP **13** (2006).

Nonlinear Photon–Plasmon Interactions

- Photons get trapped into quantum electron holes. The governing dynamical equations are the Schrödinger equations for the photons and plasmons, which are respectively

$$2i\Omega_0\left(\frac{\partial}{\partial t}+V_g\frac{\partial}{\partial x}\right)A_\perp+\frac{\partial^2 A_\perp}{\partial x^2}-\left(\frac{|\psi|^2}{\sqrt{1+|A_\perp|^2}}-1\right)A_\perp=0,$$

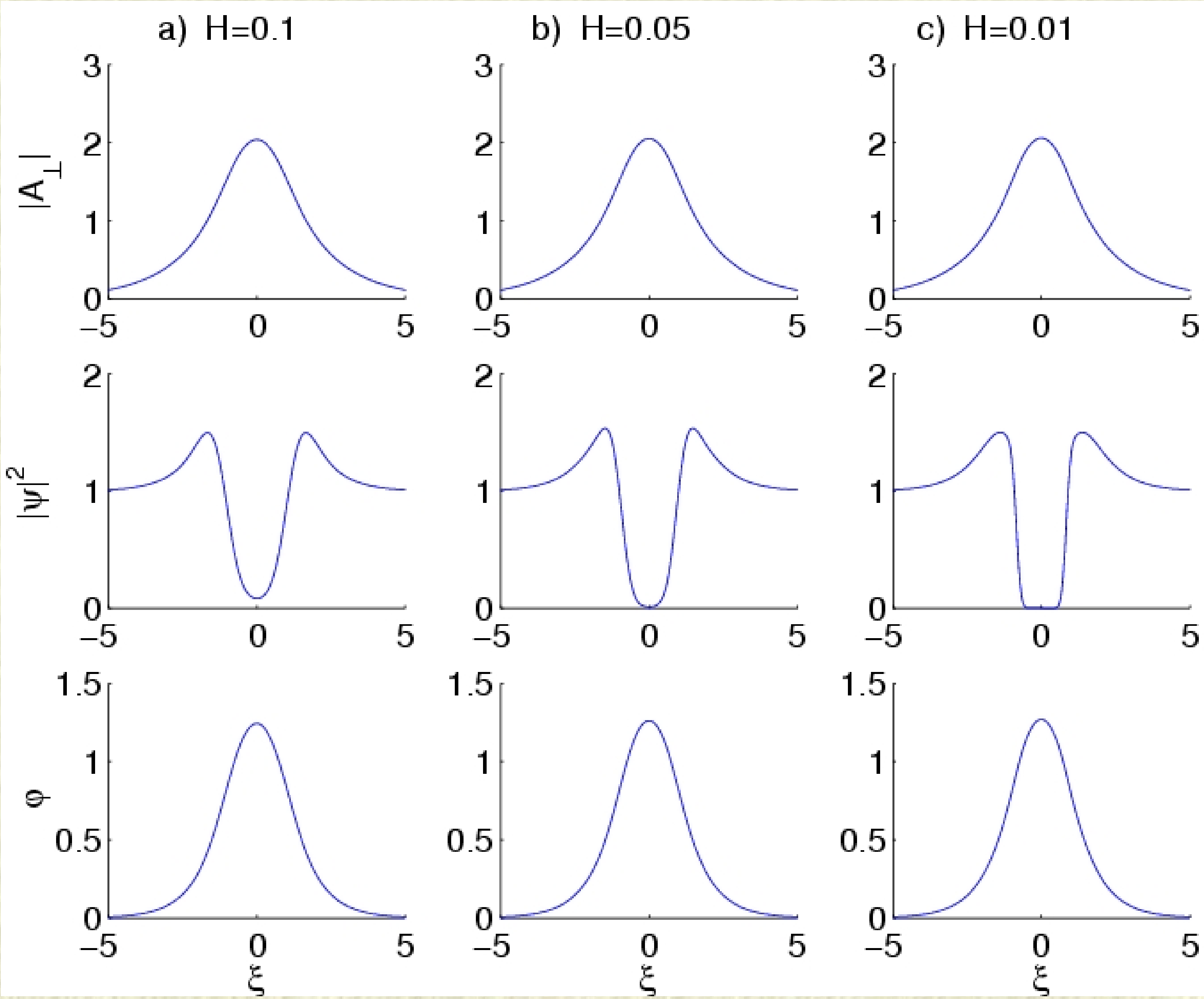
and

$$iH\frac{\partial\psi}{\partial t}+\frac{H^2}{2}\frac{\partial^2\psi}{\partial x^2}+(\varphi-\sqrt{1+|A_\perp|^2}+1)\psi=0,$$

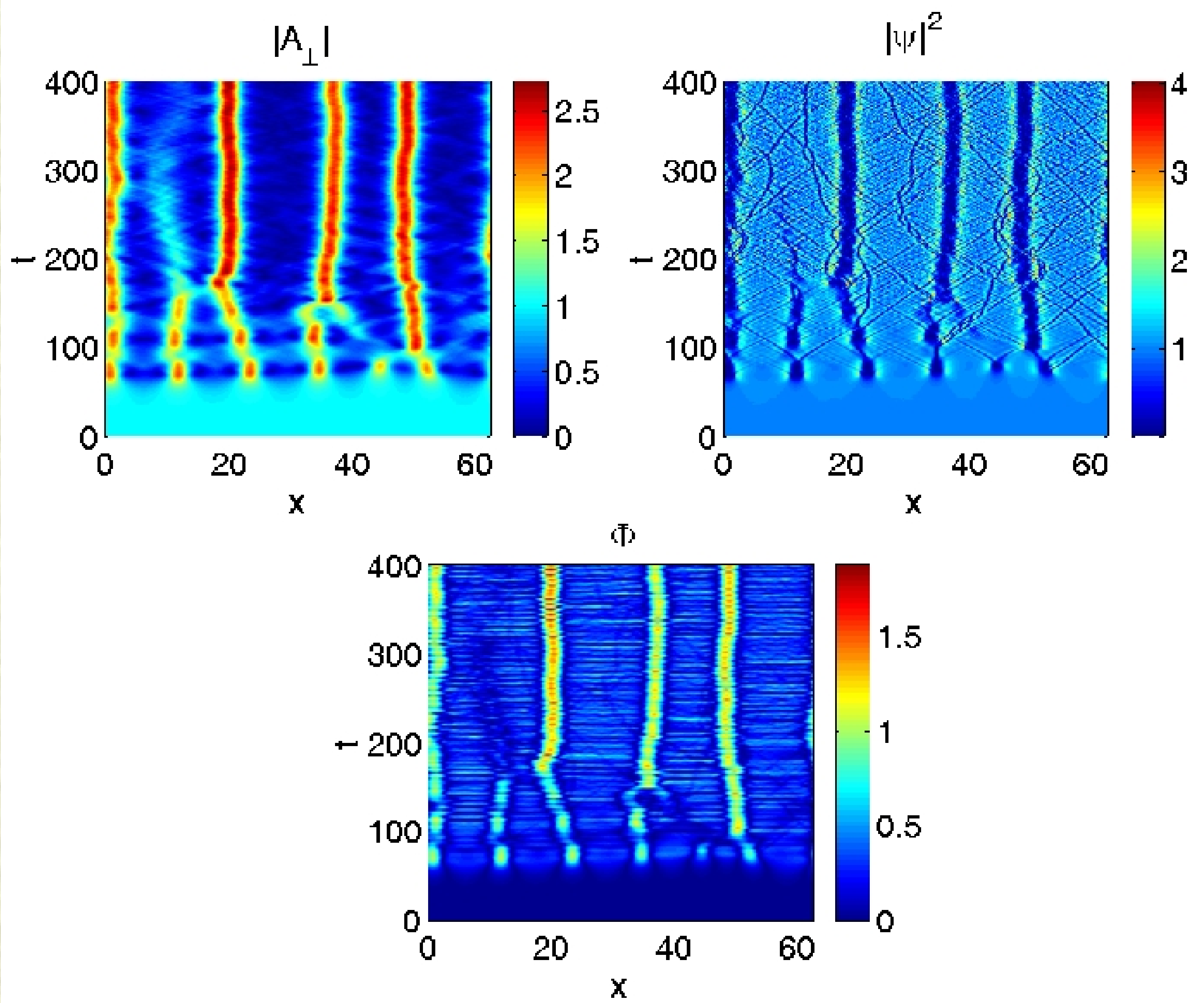
where the electrostatic potential follows the Poisson equation

$$\frac{\partial^2\varphi}{\partial x^2}=|\psi|^2-1$$

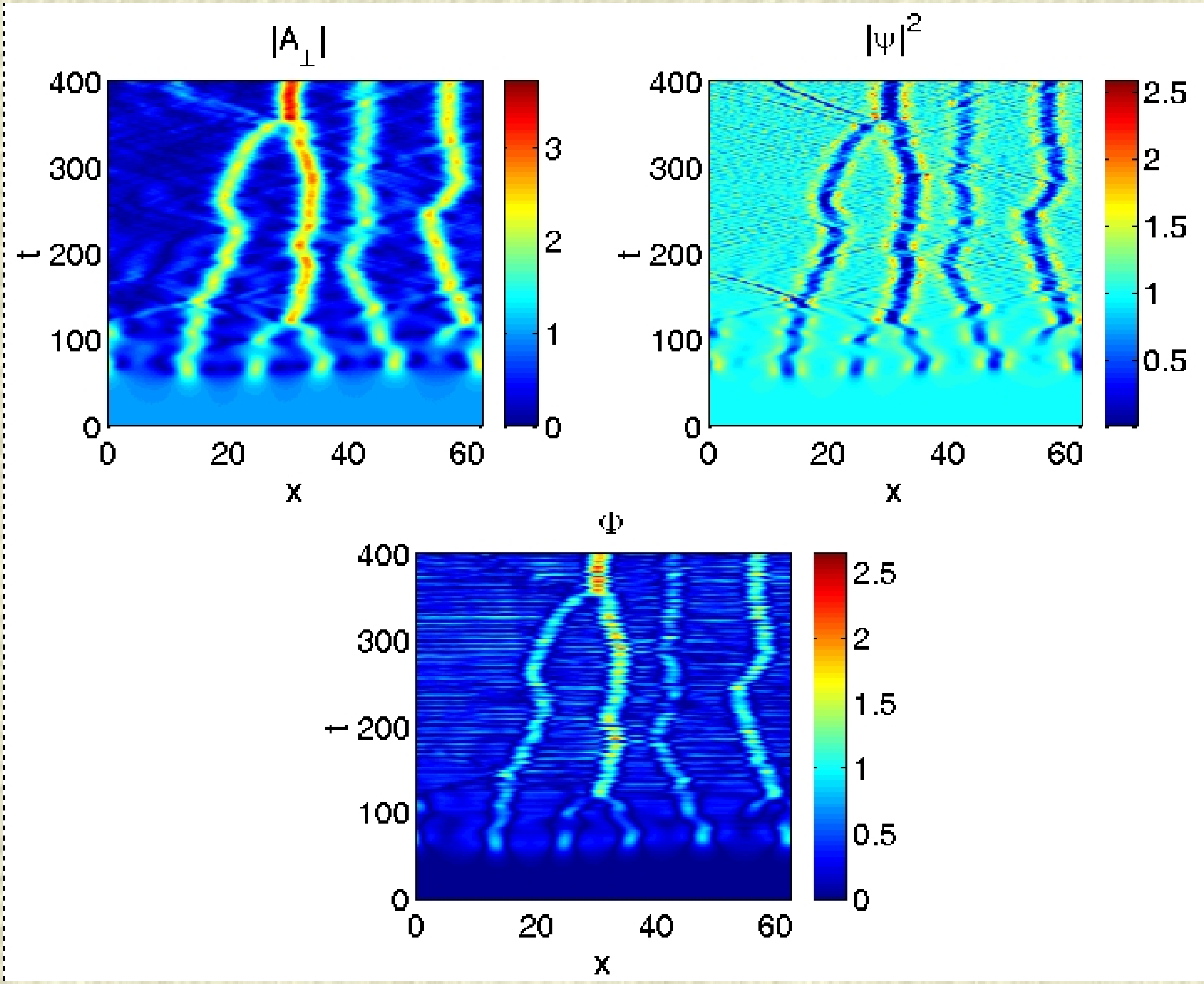
Localized Excitations for Different H



Collapse of Photons into Solitary Structures, $H = 0.1$.



Collapse of Photons into Solitary Structures, $H = 0.5$.



Summary & Conclusions

- We have discussed the properties of dense quantum plasmas.
- Provided the appropriate quantum models
- Discussed new aspects of quantum plasmonic fluid turbulence
- Presented localized quantum plasmonic excitations in the form of quantum electron holes and 2D quantum electron vortices
- Discussed the parametric amplification of quantum ES modes by photons
- The present results may be useful for information transfer at nanoscales.