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MODIFIED ZAKHAROV EQUATIONS FOR NON-LINEAR MODE COUPLING

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Abstract

The piezoelectric effect in some semiconductors have been discovered and studied in the past few decades. In most of the piezoelectric semiconductors, the effect of longitudinal electric field is observed which may cause the linear normal modes (Langmuir mode and acoustic mode) to interact. This interaction of modes has been studied employing a two time scale theory due to the mass difference between electrons and ions which makes it an inherently nonlinear problem. Such formulation of two time scale hydrodynamic model lead to Zakharov equations whose soliton solution is discussed.

Introduction

In the second half of 20th century, a substantial advancement in the field of solid state physics was brought by understanding that many physical properties of solids can be explained by elementary excitations supported by that given material. Since all the atoms in a lattice are bounded with each other by binding forces so any local perturbation can travel to other parts of the material. The spectrum of frequencies and wavelengths permitted in a lattice depends on its nature and binding forces. The coherent lattice vibrations are called phonons. If the lattice has at least two different atoms in its unit cell it exhibit piezoelectric behavior i.e. applied voltage can produce stress or electricity can be produced by applying stress. The plane wave solutions for material displacements and internal electric fields both satisfy the Maxwell's equations and the mechanical-piezoelectric equations of state. For a piezoelectric semiconductor, a part of the mechanical energy of sound waves is converted into electrical energy and electric field with its transverse and longitudinal components. The longitudinal electric field which is electrostatic nature has an effect in most piezoelectric media. So we may expect a strong coupling between these two distinct modes in a composite media.

Mathematical Model

The Hydrodynamic model has been widely used to analyze the flow of electrons in semiconductors media. The one dimensional governing equations include the following relations:

$$(1)\quad \frac{\partial n_{i,e}}{\partial t} + \frac{\partial (n_{i,e} v_{i,e})}{\partial x} = 0$$

$$(2)\quad \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = -\frac{e}{m_e}E - \frac{1}{m_e n_e}\frac{\partial P_e}{\partial x} + \frac{e^2}{4m_e^2}\frac{\partial |E|^2}{\partial x}$$

$$(3)\quad \rho_i \frac{\partial^2 u_i}{\partial t^2} - c\frac{\partial^2 u_i}{\partial x^2} = -\beta \frac{\partial E}{\partial x}$$

$$(4)\quad \frac{\partial E}{\partial x} = -\frac{e}{\varepsilon_0} n_e + \frac{\beta}{\varepsilon_0}\frac{\partial^2 u_i}{\partial x^2}$$

The Eqs (3) and (4) are elastic wave equation for ion displacement and modified Poisson's Equations derived from the mechanical-piezoelectric equations of state. The subscripts i,e denote the ions and electrons. Since we are analyzing the interaction between the electron electrostatic mode (Langmuir mode) and ion acoustic mode in a composite medium so we split all the fluid variables into slow(l) and high(h) frequency components. By doing so we arrive at the set of modified Zakharov Equations in the presence of piezoelectricity:

$$(5)\quad i\frac{\partial E}{\partial t} + \frac{1}{2}\frac{v_T^2}{\omega_{pe}^2}\frac{\partial^2 E}{\partial x^2} = \frac{\omega_{pe}}{2n_0} n_l E$$

$$(6)\quad \frac{\partial^2 n_l}{\partial t^2} + \omega_{pe}^2 n_l + \frac{\beta\omega_{pe}^2}{e}\frac{\partial^2 u_i}{\partial x^2} - v_T^2\frac{\partial^2 n_l}{\partial x^2} - \frac{\varepsilon_0}{4m_e}\frac{\partial^2 |E|^2}{\partial x^2} = 0$$

$$(7)\quad \frac{\partial^2 u_i}{\partial t^2} - \left(c_s^2 + \frac{\beta^2}{\varepsilon_0 \rho_i}\right)\frac{\partial^2 u_i}{\partial x^2} = \frac{\beta e}{\varepsilon_0 \rho_i} n_l$$

Results: The Zakharov equations admits the soliton solutions in static limit given by nonlinear Schrodinger equation. Considering the stationary solutions in a co-moving frame $\xi = x - v\,t$ and $(v^2 - v_T^2)\frac{\partial^2}{\partial \xi^2} \ll \omega_{pe}^2 + \frac{\omega_{pe}^2\beta^2}{(v^2\varepsilon_0\rho_i - cs^2\varepsilon_0\rho_i - \beta^2)}$, we get

$$(8)\quad \left[\omega_{pe}^2 + \frac{\omega_{pe}^2\beta^2}{(v^2\varepsilon_0\rho_i - cs^2\varepsilon_0\rho_i - \beta^2)}\right] n_l = \frac{\varepsilon_0}{4m_e}\frac{\partial^2 |E|^2}{\partial \xi^2}$$

It gives a standard cusp soliton.

Applications and Future Work

This work is useful is demonstrating the nonlinear interactions with in a crystal lattice which explains the nature of metals, semimetals and semiconductors. With the great degree of miniaturization of electronic components, the electron density in semiconductors exhibit quantum effects. So, this study can also be extended to quantum regime to work out the quantum corrections in these modified Zakharov equations.

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