Islamabad, March '04

Electroweak Interactions in the SM and Beyond

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A short course on the EW Theory

We start from the basic principles and formalism (a fast recall).
Then we go to present status and challenges

Content

- Formalism of gauge theories
- The SU(2)xU(1) symmetric lagrangian
- The symmetry breaking sector
- Beyond tree level
- Precision tests
- Problems of the SM
- Beyond the SM

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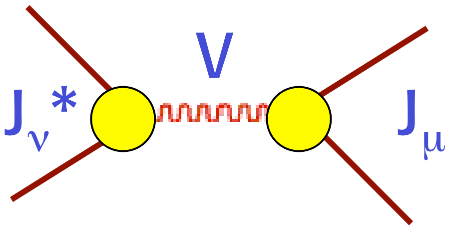
Gauge theories broken by the Higgs mechanism are renormalisable 't Hooft, Veltman

Masses are given to W, Z and fermions while gauge Ward identities and current conservation remain valid.

Essential for renormalisation!

e.g. massive V propagator (V=W,Z)



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}}{q^2 - m_W^2}$$

Bad UV behaviour

But current conservation $q_\mu J^\mu = 0$ dumps it

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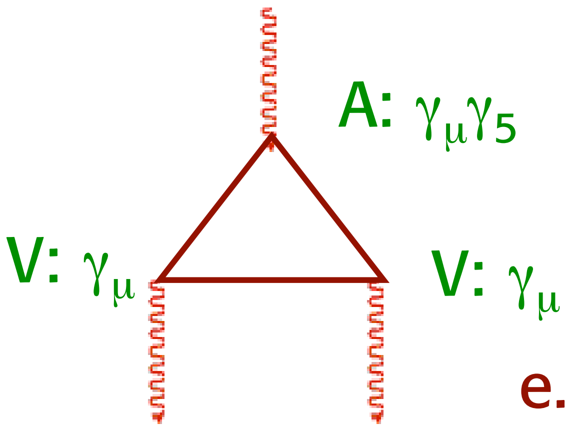
Current conservation crucial for renormalisation

But beware of chiral anomalies — Adler, Bell, Jackiw

A remarkable cancellation occurs — Bouchiat, Iliopoulos, Meyer



A V V

We need $Tr[t^3(t^3-2Qs^2_W)(t^3-2Qs^2_W)]=0$

In fact it is true! For each family

u d e ν

e.g. $Tr[t^3Q^2]= \frac{1}{2}\cdot 3\cdot\frac{4}{9}-\frac{1}{2}\cdot 3\cdot\frac{1}{9}-\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 0 =0$

colour

Similarly for $Tr[t^3t^3Q]= Tr[t^3t^3t^3]=0$

Great!! But why?? Grand unification? SU(5): $5 \to [ddde^+\nu]$

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Anomaly

In QFT when a symmetry of the classical theory is broken by quantisation, regularisation and renormalisation

Examples

- Scale A. -> Breaking of scale inv. due to reg./ren. that introduces a mass scale (cut-off, subtraction point or....)
  massless QED, QCD

- Axial A. -> Breaking of chiral symmetry $\psi' = \exp(i\gamma_5\theta)\psi$ due to a clash of reg./ren. with gauge inv.

$$\boxed{\partial_\mu j_5^\mu = \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}}$$

$$\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Important for $\pi^0 \to \gamma\gamma$, polarized DIS,....

Beyond tree level: radiative corrections

From the tree level relations $\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}$ and $g^2 s_W^2 = e^2 = 4\pi\alpha$

$$\Rightarrow \quad s_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F} \cdot \frac{1}{m_W^2}$$

$$s_W^2 \equiv sin^2\theta_W$$

Combining with $\frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \rho_0 = 1 \quad \Rightarrow \quad s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$

one obtains: $\left(1 - \frac{m_W^2}{m_Z^2}\right) \cdot m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F}$

Large pure QED effect

With radiative corr's: $\left(1 - \frac{m_W^2}{m_Z^2}\right) \cdot m_W^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F} \cdot \frac{1}{1 - \Delta r_W}$

$$\frac{m_W^2}{m_Z^2 \cos^2\theta_W} = (1 + \Delta\rho_m)$$

Depends on def. of $\sin^2\theta_W$ beyond tree level

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$\sin^2\theta_W$ is usually defined from the Z->$\mu\mu$ vertex:

$$\frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu(g_V^f - g_A^f\gamma_5)\psi Z^\mu$$

$$\begin{cases} g_A^f = \pm\frac{1}{2} \\ g_V^f/g_A^f = 1-4|Q^f|\sin^2\theta_W \end{cases} \Rightarrow \begin{array}{l} g_A^{\mu 2} \equiv \frac{1}{4}(1+\Delta\rho) \\ g_V^\mu/g_A^\mu \equiv 1-4\sin^2\theta_{eff} \end{array}$$

$\sin^2\theta_{eff}$ differs from s^2_0 defined as:

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F}\cdot\frac{1}{m_Z^2}$$

{ Recall: $\left(1-\frac{m_W^2}{m_Z^2}\right)\cdot m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F}$ }

by a rad. corr.:

$\sin^2\theta_{eff} = (1+\Delta k')\, s^2_0$

Δr_W, $\Delta\rho$, $\Delta k'$ at one loop all contain terms of order:
$G_F m^2_W[1,\ m^2_t/m^2_W,\ \log(m^2_H/m^2_W)]$

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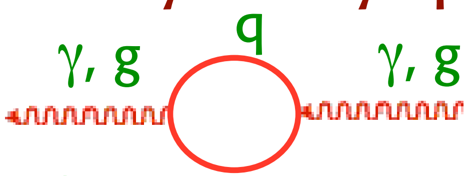
m_t, m_H do not decouple!

In the standard EW theory heavy loops do not decouple

Decoupling: for M -> infinity we can drop diagrams with internal M lines

For example: running of α, α_s not affected by heavy quarks



Conditions for decoupling: Applequist, Carazzone

- The theory with no M should still be renorm.
- Couplings should not blow up with M -> infinity

In QED, QCD one can decouple m_t

In EW sector one cannot decouple m_t, m_H:

* breaking of gauge inv. (t-b doublet, $G_F(m^2_t - m^2_b)$)

* couplings of longitudinal W, Z grow with masses (Higgs mechanism)

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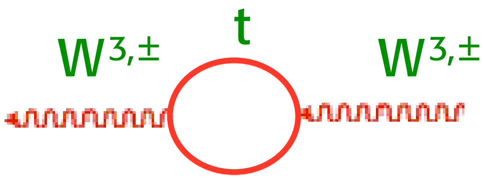
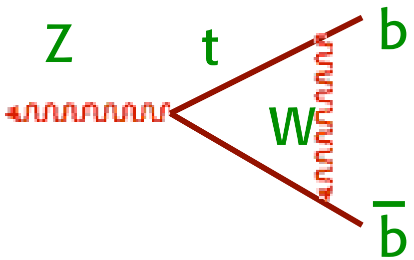
One-loop diagrams leading to $G_F m^2_t$ terms:

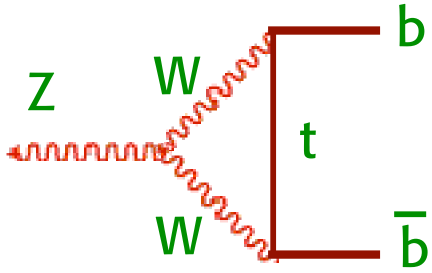


Δr_W, $\Delta\rho$, $\Delta k'$

Enough sensitivity to correctly estimate m_t from rad. corr.s

Note: self-energies universal. All heavy particles enter.





ε_b

At one-loop $G_F m^2_H$ terms are absent. While $m_t >> m_b$ directly breaks SU(2), Higgs couplings are invariant in lowest order. At two-loops $(G_F m^2_H)^2$ terms are present

Veltman, Van der Bij

This is unfortunate: small sensitivity of rad. corr. to m_H -> $G_F m^2_W \log(m^2_H/m^2_W)$

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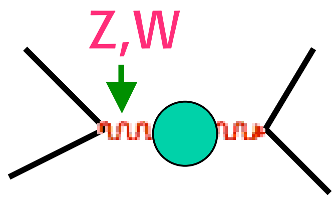
EW DATA and New Physics

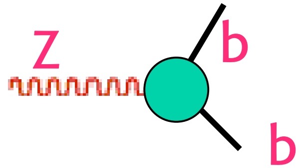
For an analysis of the data beyond the SM we use the ε formalism GA, R.Barbieri, F.Caravaglios, S. Jadach

One introduces ε_1, ε_2, ε_3, ε_b such that:

- Focus on pure weak rad. correct's, i.e. vanish in limit of tree level SM + pure QED and/or QCD correct's [a good first approximation to the data]

- Are sensitive to vacuum pol. ε_1, ε_2, ε_3 → and Z->bb vertex corr.s (but also include non oblique terms) ε_b →





- Can be measured from the data with no reference to m_t and m_H (as opposed to S, T, U -> ε_3, ε_1, ε_2)

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One starts from a set of defining observables:

$$O_i = m_W/m_Z, \quad \Gamma_\mu, \quad A^\mu_{FB}, \quad R_b$$

$$\varepsilon_1 \qquad \varepsilon_3$$

$$\varepsilon_2 \qquad\qquad \varepsilon_b$$

$$O_i[\varepsilon_k] = O_i^{\text{"Born"}}[1 + A_{ik}\,\varepsilon_k + \ldots]$$

$O_i^{\text{"Born"}}$ includes pure QED and/or QCD corr's.

A_{ik} is independent of m_t and m_H

Assuming lepton universality: Γ_μ, A^μ_{FB} --> Γ_l, A^l_{FB}

To test lepton-hadron universality one can add Γ_Z, σ_h, R_l to Γ_l etc.

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$\varepsilon_1, \varepsilon_2$ and ε_3 are related to Δr_W, $\Delta\rho$ and $\Delta k'$

Large $G_F m^2_t$ terms in Δr_W, $\Delta\rho$ and $\Delta k'$ $\Rightarrow$

$$\Delta r_W \sim \frac{c_W^2 - s_W^2}{s_W^2}\Delta k' \sim -\frac{c_W^2}{s_W^2}\Delta\rho$$

$$\boxed{\begin{aligned} \varepsilon_1 &\equiv \Delta\rho \\ \varepsilon_2 &\equiv c_W^2\Delta\rho + \frac{s_W^2}{c_W^2 - s_W^2}\Delta r_W - 2 s_W^2 \Delta k' \\ \varepsilon_3 &\equiv c_W^2 \Delta\rho + (c_W^2 - s_W^2)\Delta k' \end{aligned}}$$

$$\Delta\rho \sim \frac{3 G_F m_t^2}{8\pi^2\sqrt{2}}$$

- Large $G_F m^2_t$ terms are only in ε_1
- Main m_H sensitivity in ε_3
- m_W sensitivity through Δr_W in ε_2

In addition ε_b arises from the Z->bb vertex

$$\varepsilon_b \sim -\frac{G_F m_t^2}{4\pi^2\sqrt{2}}$$

Relation with S, T, U: the shifts from new physics are proportional $\Delta S \sim \Delta\varepsilon_3$, $\Delta T \sim \Delta\varepsilon_1$, $\Delta U \sim \Delta\varepsilon_2$

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The EWWG gives (summer '03):

$\varepsilon_1 = 5.4 \pm 1.0 \; 10^{-3}$

$\varepsilon_2 = -\; 9.7 \pm 1.2 \; 10^{-3}$

$\varepsilon_3 = 5.25 \pm 0.95 \; 10^{-3}$

$\varepsilon_b = -\; 4.7 \pm 1.6 \; 10^{-3}$

Non-degenerate
much larger shift of ε_1

For comparison:
a mass degenerate fermion multiplet gives

$$\Delta\varepsilon_3 = N_C \frac{G_F m_W^2}{8\pi^2\sqrt{2}} \cdot \frac{4}{3}[T_{3L} - T_{3R}]^2$$

For each member of the multiplet

One chiral quark doublet (either L or R):

$$\Delta\varepsilon_3 = +\; 1.4 \; 10^{-3}$$

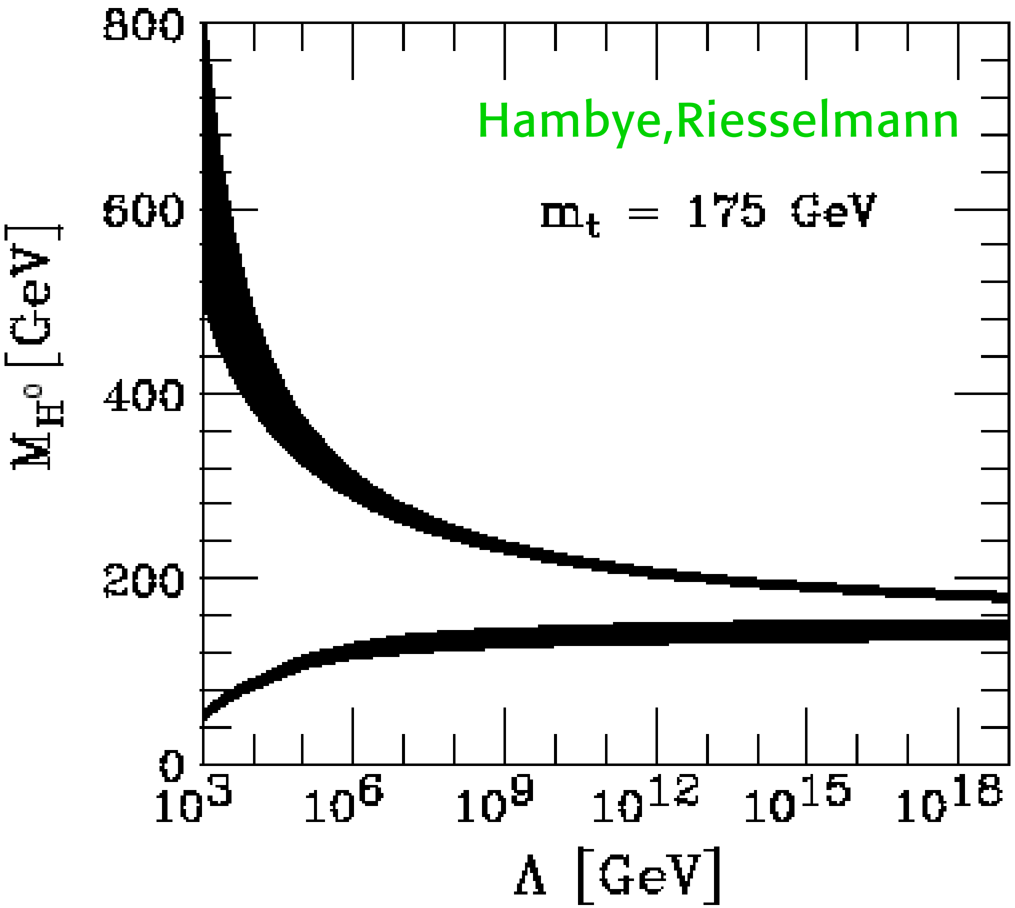
(Note that ε_3 if anything is low!)

Theoretical bounds on the SM Higgs mass

Λ: scale of new physics beyond the SM

Upper limit: No Landau pole up to Λ

Lower limit: Vacuum (meta)stability



If the SM would be valid up to M_{GUT}, M_{Pl} then m_H would be limited in a small range

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Higgs potential

"Wrong" sign

Classic: $V[\phi] = -\mu^2\phi^2 + \lambda\phi^4$ $\mu^2>0,\ \lambda>0$

$$\phi \Rightarrow v + \frac{H}{\sqrt{2}} \quad \Longrightarrow \quad v^2 = \frac{\mu^2}{2\lambda} = \frac{m_H^2}{4\lambda}$$

Quantum loops: $\lambda\phi^4 \Rightarrow \lambda\phi^4\left(1 + \gamma \ln\frac{\phi^2}{\Lambda^2} + \ldots\right) \xrightarrow{\text{RG}} \lambda(\Lambda)\phi'^4(\Lambda)$

(Ren. group improved pert. th)

$$\phi' = [\exp\int\gamma(t)dt]\phi$$

Running coupling

$t=\ln\Lambda/v$ h_t=top Yukawa

$$\frac{d\lambda(t)}{dt} = \beta_\lambda(t) = const[\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + small]$$

Initial conditions (at Λ=v) $\lambda_0 = \frac{m_H^2}{4v^2}$ and $h_{0t} = \frac{m_t}{v}$

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Running coupling $t = \ln\Lambda/v$ h_t=top Yukawa

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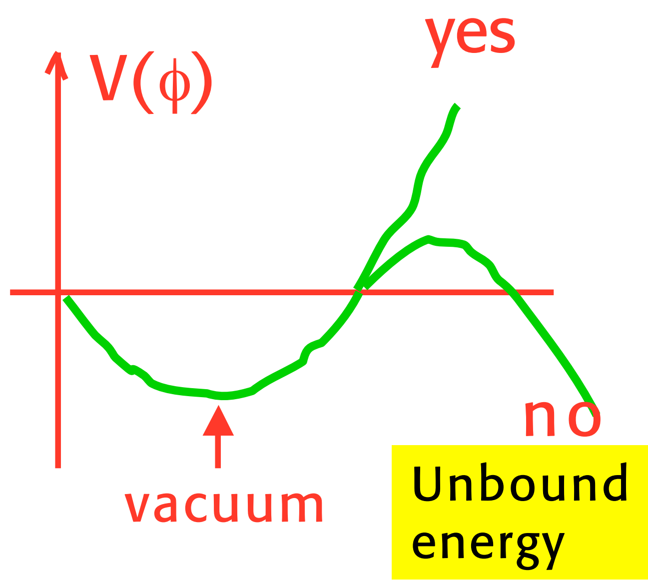
Initial conditions (at Λ=v) $\lambda_0 = \frac{m_H^2}{4v^2}$ and $h_{0t} = \frac{m_t}{v}$

Too small m_H? h_t wins, $\lambda(t)$ decreases.
But $\lambda(t)$ must be >0 below Λ for the vacuum to be stable

⟹ m_H ~135 GeV if $\Lambda \sim M_{GUT}$

(or at least metastable with lifetime $\tau > \tau_{Universe}$)



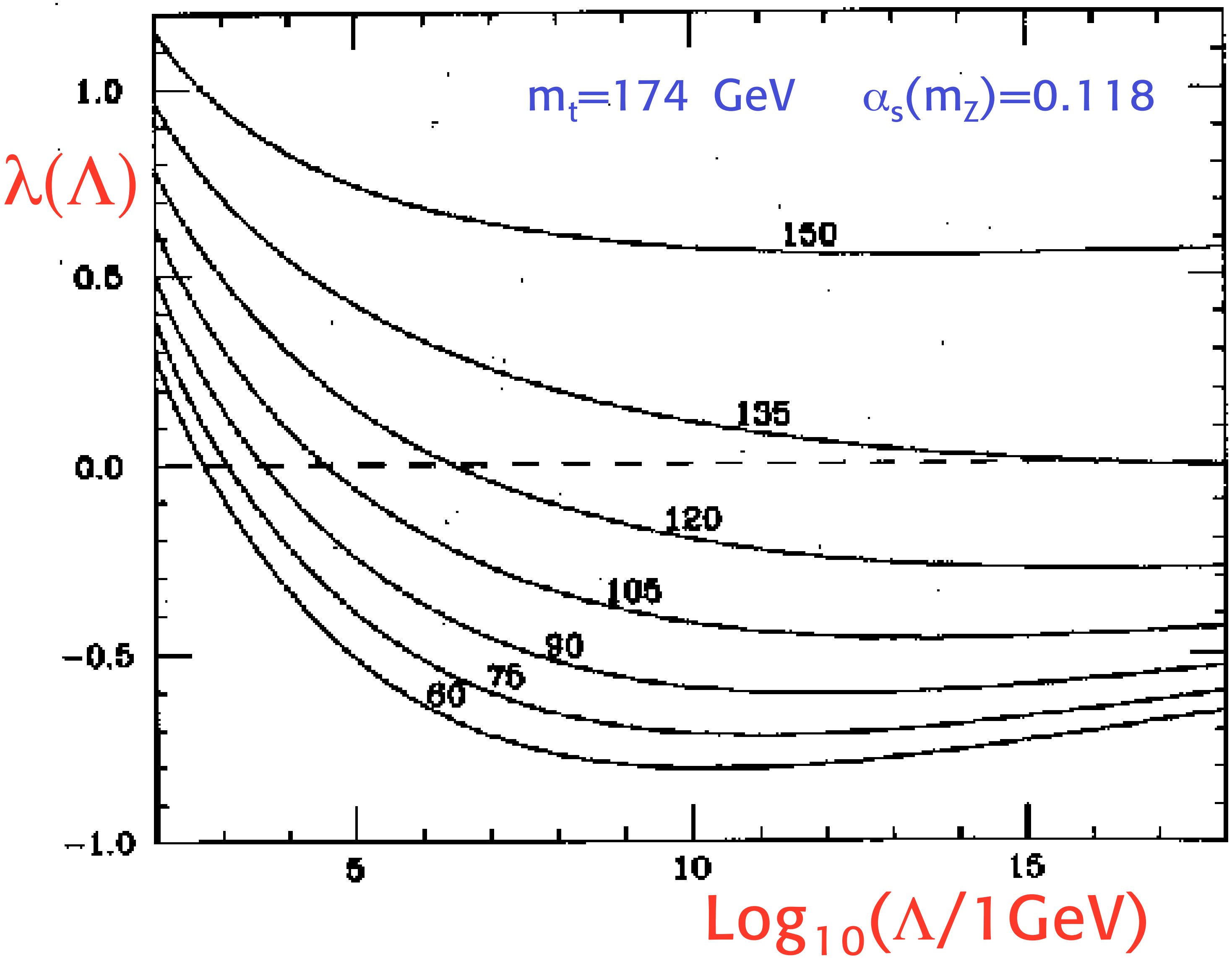
Cabibbo et al, Sher, Altarelli, Isidori

stability $m_H(\mathrm{GeV}) > 133 + 2.0\,[m_t(\mathrm{GeV}) - (175 \pm 2)] - 1.6 \left[\frac{\alpha_s(m_Z) - 0.118}{0.002}\right]$

metastability $m_H(\mathrm{GeV}) > 117 + 2.9\,[m_t(\mathrm{GeV}) - (175 \pm 2)] - 2.5 \left[\frac{\alpha_s(m_Z) - 0.118}{0.002}\right]$

Isidori, Ridolfi, Strumia

Altarelli,Isidori



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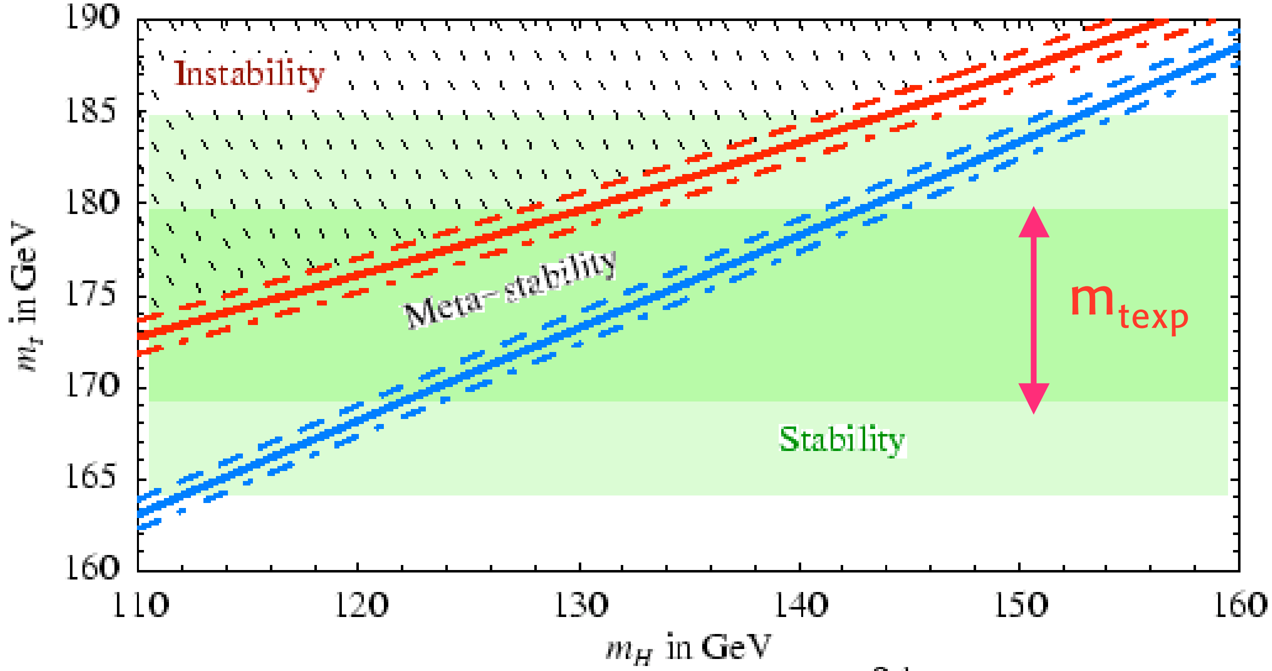
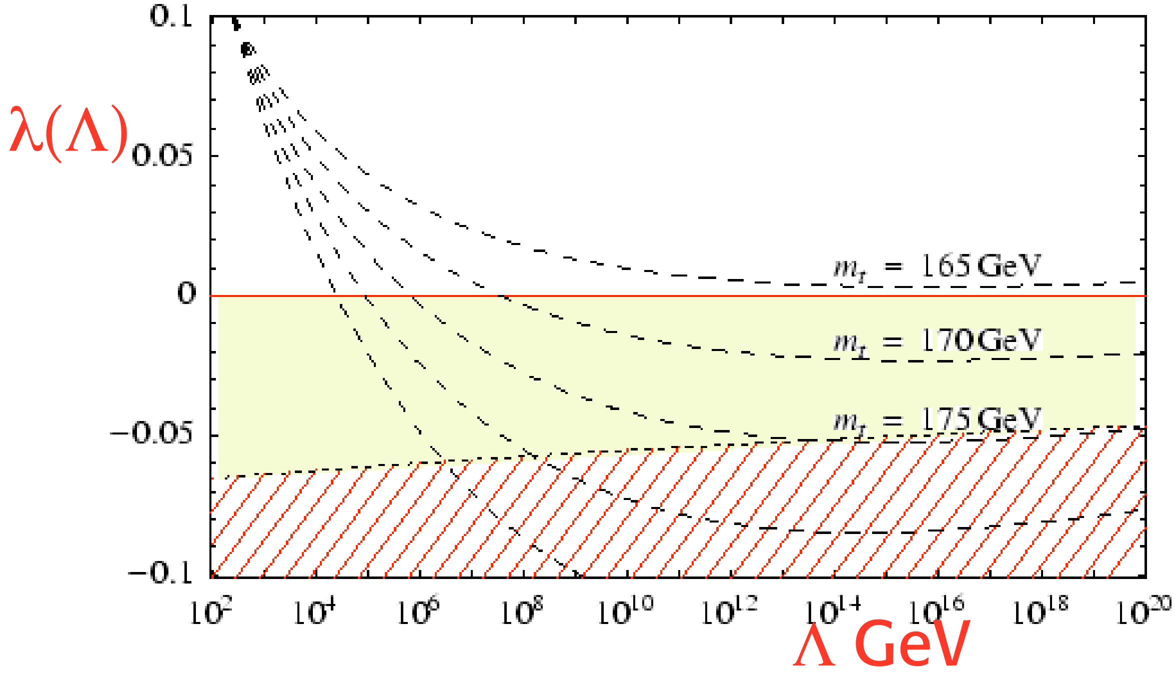


Isidori, Ridolfi, Strumia



Here m_H=115 GeV
$\alpha_s(m_Z)$=0.118

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Running coupling $t = \ln \Lambda / v$ h_t=top Yukawa

$$\frac{d\lambda(t)}{dt} = \beta_\lambda(t) = const[\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + small]$$

b

Initial conditions (at Λ=v) $\lambda_0 = \frac{m_H^2}{4v^2}$ and $h_{0t} = \frac{m_t}{v}$

Too large m_H? λ^2 wins, $\lambda(t)$ increases.

$$\lambda(t) \sim \frac{\lambda_0}{1 - b\lambda_0 t}$$

Landau pole

The upper limit on m_H is obtained by requiring that no Landau pole occurs below Λ

m_H ~180 GeV if $\Lambda \sim M_{GUT}$
~ 600-800 GeV if $\Lambda \sim o(TeV)$

Rather than a bound says where non pert effects are important

Caution: near the pole pert. theory inadequate.
Simulations on the lattice appear to confirm the bound
Kuti et al, Hasenfratz et al, Heller et al

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Precision tests of the SM

Input parameters:

α, G_F, m_Z, m_{flight}, $\alpha_s(m_Z)$, m_t, m_H

in practice replaced by $\alpha(m_Z)$

Some are well known

α, G_F, m_Z

Some are less precise

$\alpha(m_Z)$, $\alpha_s(m_Z)$, m_t

m_H is unknown

Computed rad corr:

- complete 1-loop diagrams
- ren group improvements (large logs)
- Dyson resumm's of some large terms
- selected dominant 2-loop corr's.

  eg $G_F m_t^2 \alpha_s$, $G_F^2 m_t^4$, $G_F^2 m_H^2$....

Precision data: Γ_Z, R_h, σ_h, R_b, A^l_{FB}, A^τ_{pol}, A_{LR}, A^b_{FB}, m_W, Q_{APV}....

Output: check consistency of SM, constrain m_H...

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Status of the SM Higgs fit

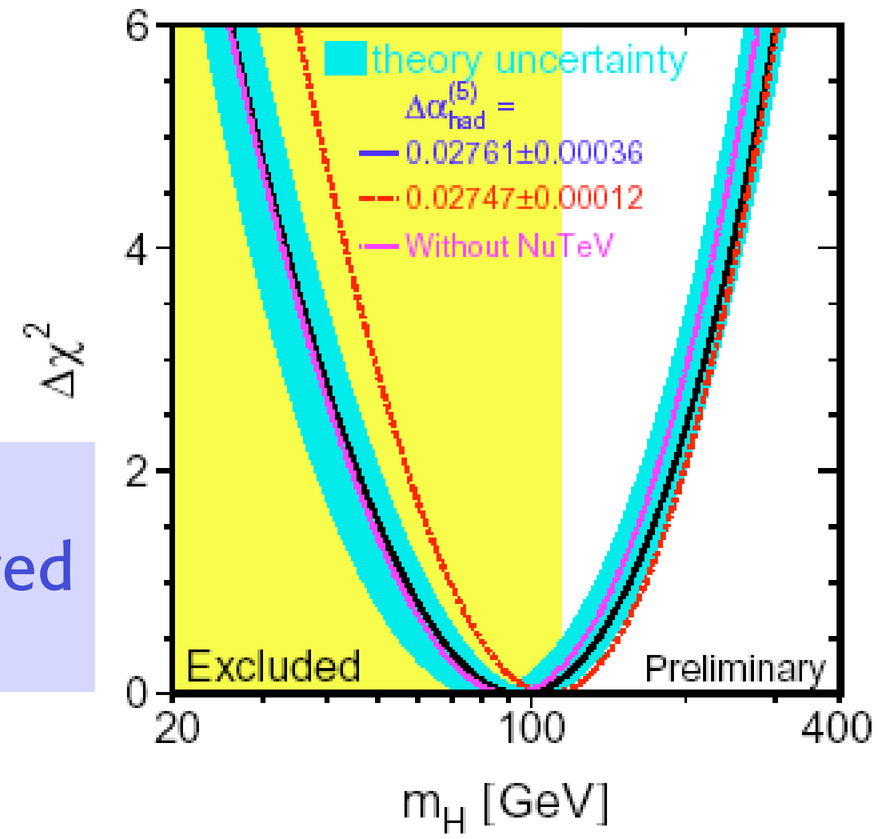
Summer'03

Sensitive to log m_H

Rad Corr.s ->

$\log_{10} m_H(\text{GeV}) = 1.96 \pm 0.21$

This is a great triumph for the SM: right in the narrow allowed window $\log_{10} m_H \sim 2 - 3$



Direct search: m_H> 114 GeV

	All Z pole	All data	All but NuTeV
m_t (GeV)	$171.5^{+11.9}_{-9.4}$	$174.3^{+4.5}_{-4.4}$	$175.3^{+4.4}_{-4.3}$
m_H (GeV)	89^{+122}_{-45}	96^{+60}_{-38}	91^{+55}_{-36}
$\alpha_S(M_Z^2)$	0.1187 ± 0.0027	0.1186 ± 0.0027	0.1185 ± 0.0027
χ^2/dof (P)	14.7/10(14.3%)	25.4/15(4.5%)	16.7/14(27.5%)

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$\log_{10} m_H \sim 2$ is a very important result

Drop H from SM -> renorm. lost -> divergences -> cut-off Λ

$\log m_H \to \log\Lambda$ + const

Any alternative mechanism amounts to change the prediction of finite terms.

The most sensitive quantities to $\log m_H$ are $\varepsilon_1 \sim \Delta\rho$ and ε_3:

$\log_{10} m_H \sim 2$ means that $f_{1,3}$ are compatible with the SM prediction

$$\varepsilon_1 = -\underbrace{\frac{3 G_F m_W^2}{4\pi^2\sqrt{2}}}_{-1.2\ 10^{-3}} \mathrm{tg}^2\theta_W \left[\log\frac{m_H}{m_Z} + f_1\right]$$

$$\varepsilon_3 = \underbrace{\frac{G_F m_W^2}{12\pi^2\sqrt{2}}}_{0.45\ 10^{-3}} \left[\log\frac{m_H}{m_Z} + f_3\right]$$

New physics can change the bound on m_H (different $f_{1,2}$)

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The EW theory: $\mathcal{L} = \mathcal{L}_{\text{symm}} + \mathcal{L}_{\text{Higgs}}$

$$\mathcal{L}_{symm} = -\frac{1}{4}[\partial_\mu W^A_\nu - \partial_\nu W^A_\mu - ig\varepsilon_{ABC}W^A_\mu W^B_\nu]^2 + -\frac{1}{4}[\partial_\mu B_\nu - \partial_\nu B_\mu]^2 + +\bar{\psi}\gamma^\mu[i\partial_\mu + gW^A_\mu t^A + g'B_\mu \frac{Y}{2}]\psi$$

$$\mathcal{L}_{Higgs} = |[\partial_\mu - igW^A_\mu t^A - ig'B_\mu \frac{Y}{2}]\phi|^2 + + V[\phi^\dagger\phi] + \bar{\psi}\Gamma\psi\phi + h.c$$

$$\psi = \psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

A chiral theory:

$$(t, Y)_R \neq (t, Y)_L$$

$$Q = t^3 + Y/2$$

with

$$V[\phi^\dagger\phi] = \mu^2(\phi^\dagger\phi)^2 + \lambda(\phi^\dagger\phi)^4$$

$\mathcal{L}_{\text{symm}}$: well tested (LEP, SLC, Tevatron…), $\mathcal{L}_{\text{Higgs}}$: ~ untested

Rad. corr's -> $m_H < 193$ GeV

but no Higgs seen: $m_H > 114.4$ GeV; ($m_H = 115$ GeV ?) LEP: 2.1σ

Only hint $m_W = m_Z \cos\theta_W \longrightarrow$ doublet Higgs

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