Heavy Flavour Physics- Rare B-Decays

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Plan of Talk

- Inclusive Radiative Decay $B \longrightarrow X_s \gamma$
- Exclusive Radiative Decays $B \longrightarrow (K^*, \rho, \omega)\gamma$
- Inclusive Semileptonic Decay $B \longrightarrow X_s \ell^+ \ell^-$
- Exclusive Semileptonic Decays $B \longrightarrow (K, K^*)\ell^+ \ell^-$
- A Model-independent Analysis of $B \longrightarrow X_s \gamma$ and $B \longrightarrow X_s \ell^+ \ell^-$
- The Decay $B_s \longrightarrow \mu^+ \mu^-$ in the SM and SUSY
- Summary

Interest in Rare B Decays

- Rare B Decays ($b \to s\gamma, b \to d\gamma, b \to s\ell^+\ell^-, ...$) are Flavour-Changing-Neutral-Current (FCNC) processes ($|\Delta B| = 1, |\Delta Q| = 0$)
- In the SM, all electrically neutral bosons (γ, Z^0, H^0, Gluons) have only Flavour-diagonal couplings. Hence, in the SM, FCNC processes are not allowed at the Tree level
- Instead, FCNC processes are governed by the GIM mechanism, which imparts them sensitivity to higher scales (m_t, m_W)
- GIM amplitudes (renormalized by QCD corrections) involve, in particular, CKM matrix elements $V_{ti}; i = d, s, b$; hence rare B-decays play an important role in the determination of these matrix elements
- FCNC processes are sensitive to physics beyond the SM, such as supersymmetry, and the BSM amplitudes can be comparable to the (tW)-part of the GIM amplitudes
- Last, but not least, Rare B-decays enjoy great attention in the ongoing and planned experimental programme in heavy quark physics (CLEO, BABAR, BELLE, CDF, D0, LHC, Super-B factory)

Rare B decays

Two inclusive rare B-decays of current experimental interest

$$\bar{B} \to X_s \gamma \quad \text{and} \quad \bar{B} \to X_s l^+ l^-$$

$X_s =$ any hadronic state with $S = -1$, containing no charmed particles

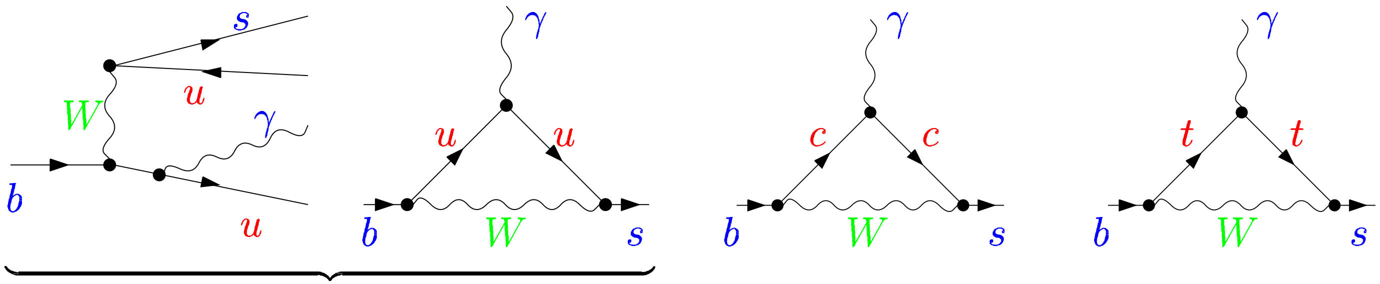
Theoretical Interest:

- Accurate measurements anticipated in near future
- Non-perturbative effects under control
- Sensitivity to new physics

Status of the NNLO perturbative calculations:

- $\bar{B} \to X_s l^+ l^-$: completed
- $\bar{B} \to X_s \gamma$: $\sim \frac{1}{3}$ way through [Misiak, Steinhauser, Greub, Haisch, Gorbahn, Schröder, Czakon,...]

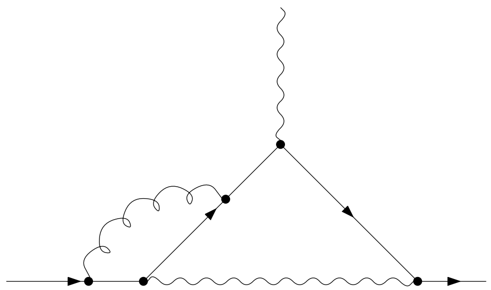
Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$$\underbrace{\sim +200\% \qquad \sim -100\%}$$

In the amplitude, after including LO QCD effects.



QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($\bar{B} \to X_s \gamma$) more than twice.

Effective field theory method is the most convenient for resummation of such large logarithms.

The effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(q,l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, \;\; l = e, \mu)$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\mathbf{\Sigma}_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, \mathbf{10} \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Structure of the SM calculations for $\bar{B} \to X_s \gamma$

$$\mathcal{H}_{\text{eff}} \sim \sum_{i=1}^{10} C_i(\mu) O_i$$

• $\mathcal{H}_{\text{eff}}$ independent of the scale μ, while $C_i(\mu)$ and $O_i(\mu)$ depend on μ
$\Longrightarrow$ Renormalization Group Equation (RGE) for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^{\mathrm{T}} C_j(\mu)$$

• γ_{ij}: anomalous dimension matrix

• Matching usually done at high scale $(\mu_0 \sim M_W, m_t)$

• Full theory and the matrix elements of the effective operators have the same large logarithms

$\mu_0 \sim O(M_W)$
$\downarrow$ RGE
$\mu_b \sim O(m_b)$: matrix elements of the operators at this scale don't have large logs; they are contained in the $C_i(\mu_b)$

• Evaluation of the on-shell amplitudes at $\mu_b \sim m_b$

Status of the SM calculations for $\bar{B} \to X_s \gamma$ (Courtesy: M. Misiak)

Matching $(\mu_0 \sim M_W, m_t)$:

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$$

$i = 1, ..., 6$:	tree	1-loop	2-loop	[Bobeth, Misiak, Urban, NPB 574 (2000) 291]
$i = 7, 8$:	1-loop	2-loop	3-loop	[Steinhauser, Misiak, hep-ph/0401041]

The 3-loop matching has less than 2% effect on BR$(\bar{B} \to X_s\gamma)$

Mixing:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 1L & 2L \\ 0 & 1L \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2L & 3L \\ 0 & 2L \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3L & 4L \\ 0 & 3L \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schröder, Czakon

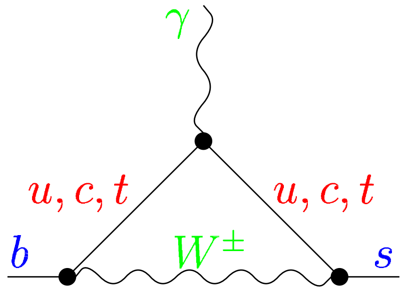
Matrix elements $(\mu_b \sim m_b)$:

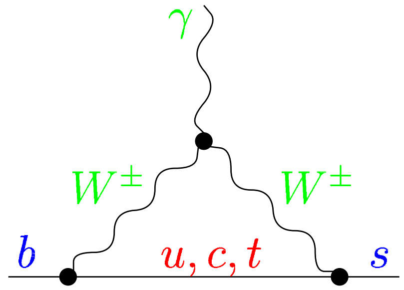
$$\langle O_i \rangle(\mu_b) = \langle O_i \rangle^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} \langle O_i \rangle^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i \rangle^{(2)}(\mu_b)$$

$i = 1, ..., 6$:	1-loop	2-loop	3-loop [Bieri, Greub, Steinhauser, hep-ph/0302051] $\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, Misiak
$i = 7, 8$:	tree	1-loop	2-loop [Greub, Hurth, Asatrian]

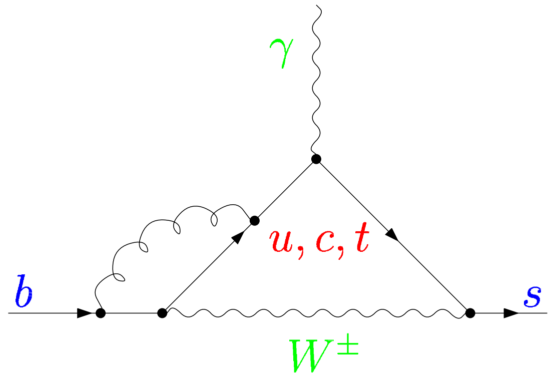
Examples of SM diagrams for the matching of $C_7(\mu_0)$:

LO:
[Inami, Lim, 1981]

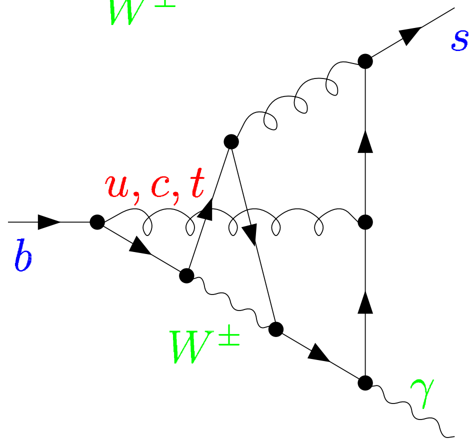




NLO:
[Adel, Yao, 1993]



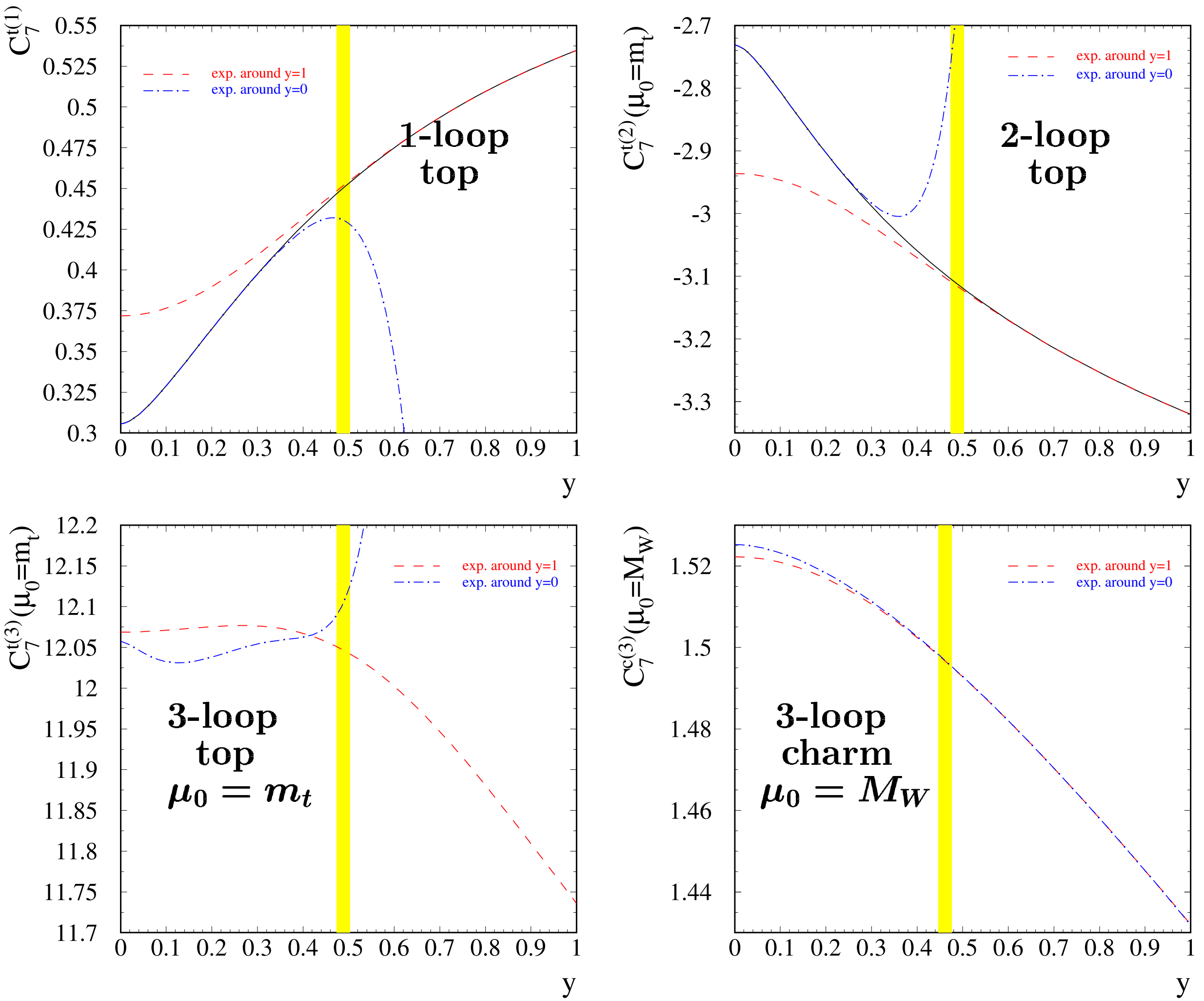
NNLO:
[Steinhauser, Misiak, 2004]



The "flavour-split" matching condition:

$$V_{ts}^* V_{tb} C_7(\mu_0) \equiv V_{ts}^* V_{tb} C_7^t(\mu_0) + \left(V_{us}^* V_{ub} + V_{cs}^* V_{cb}\right) C_7^c(\mu_0)$$

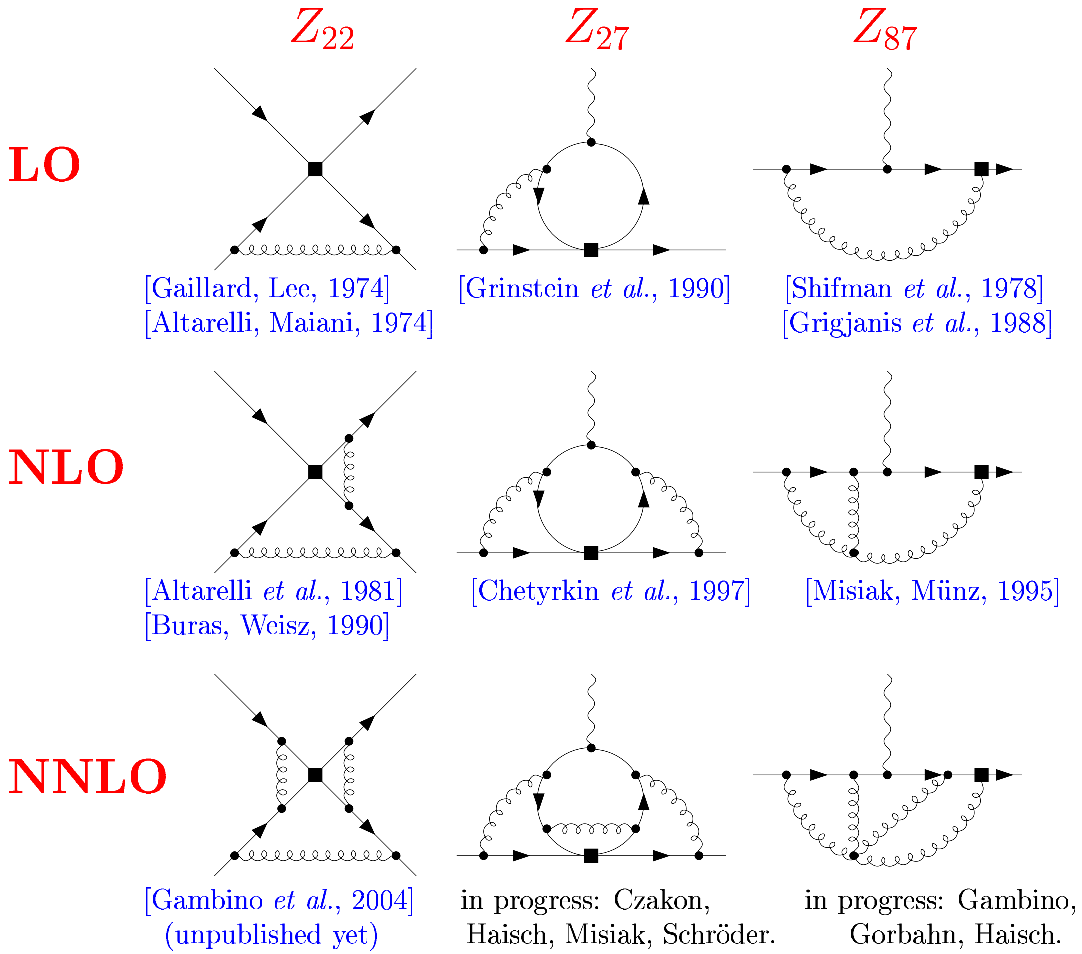
The coefficients $C_7^{Q(n)}(\mu_0)$ as functions of $y = \frac{M_W}{m_t(\mu_0)}$



Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in $b \to s\gamma$ amplitude

RGE for the Wilson coefficients $\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$

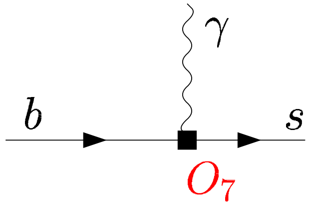
- Renormalization constants $\Longrightarrow$ γ_{ij}



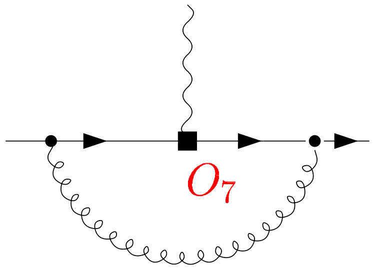
The $b \to s\gamma$ matrix elements

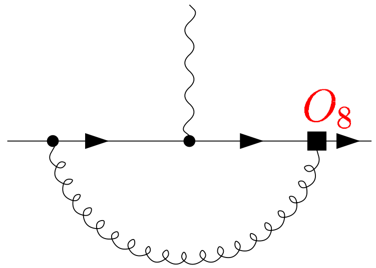
Perturbative on-shell amplitudes

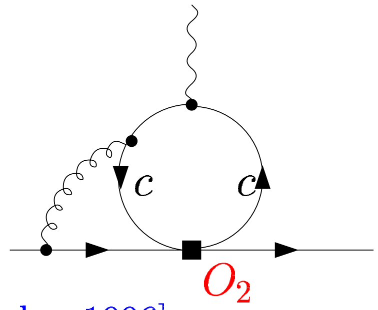
LO



NLO



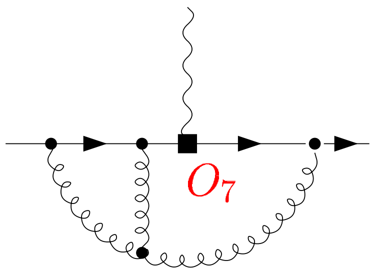


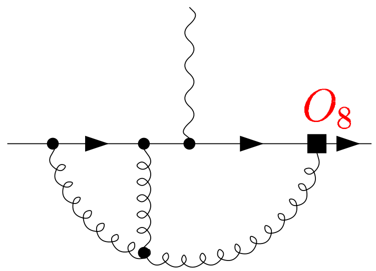


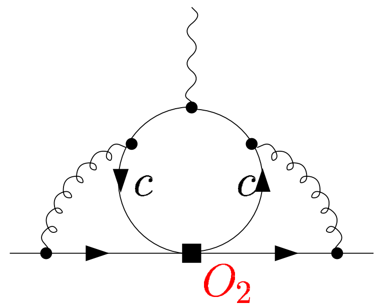
[Ali, Greub, 1991]

[Greub, Hurth, Wyler, 1996]

NNLO







in progress: Asatrian, Greub, Hurth

[Bieri *et al*, 2003] ($\mathcal{O}(\alpha_s^2 n_f)$)
in progress: Steinhauser, Misiak
(extrapolation in m_c)

E_γ-Spectrum in $B \to X_s \gamma$ in $O(\alpha_s^2)$

Melnikov and Mitov; hep-ph/0505097

- Assuming that the decay is dominated by $\mathcal{O}_7$; calculate normalized E_γ-spectrum in $O(\alpha_s^2)$ $[z = 2E_\gamma/m_b]$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz} = \delta(1-z) + (\frac{\alpha_s}{\pi})C_F F^{(1)}(z) + (\frac{\alpha_s}{\pi})^2 C_F F^{(2)}(z)$$

- Normalization $\int_0^1 \frac{1}{\Gamma}\frac{d\Gamma}{dz} dz = 1$ allows to fix the $\delta(1-z)$ term

- $O(\alpha_s)$ contribution [Greub, AA; Z. Phys. C49 ('91) 431]

$$F^{(1)}(z) = -\frac{31}{12}\delta(1-z) - \left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{7}{4}\left[\frac{1}{1-z}\right]_+ - \frac{z+1}{2}\ln(1-z) + \frac{7+z-2z^2}{4}$$

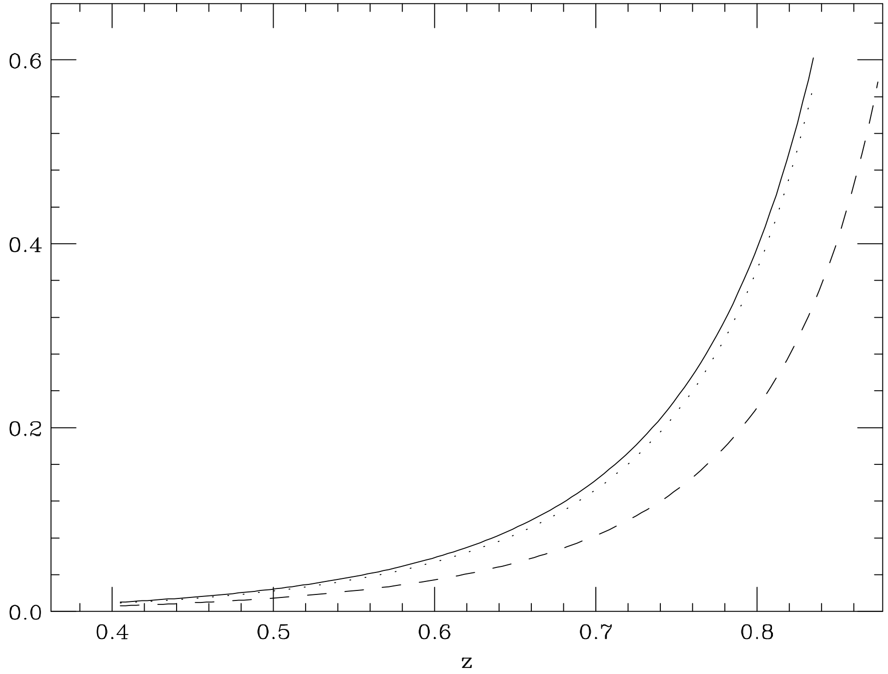
- BLM [Brodsky-Lepage-Mackenzie] corrections to $O(\alpha_s)^2\beta_0$ obtained by calculating the $O(\alpha_s)^2 n_f$ piece and making the identification $-2n_f/3 \to \beta_0$ [Ligeti, Luke, Manohar, Wise; hep-ph/9903305]

- BLM corrections summed to all orders in α_s[Benson, Bigi, Uraltsev; hep-ph/0410080]

E_γ-Spectrum in $B \to X_s \gamma$ in $O(\alpha_s^2)$ (Contd.)

Melnikov and Mitov; hep-ph/0505097

- E_γ-spectrum in $\mathcal{O}(\alpha_s^2)$ (solid), BLM (dots) and $\mathcal{O}(\alpha_s)$ (dashed)



- Effect of the non-BLM terms is about 1% for $\mu = m_b$

Non-perturbative effects in $\bar{B} \to X_s \gamma$

We need to sum the matrix elements of the effective Hamiltonian:

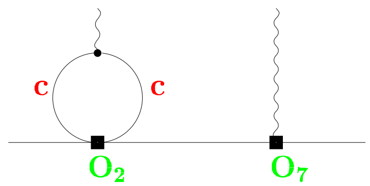
$$\Sigma_{X_s} \left| C_7 \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2 \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$$

The "77" term in the above sum can be related via optical theorem to the imaginary part of the elastic forward scattering amplitude

HQET gives us a double expansion:

$$\Sigma_{X_s} \mathrm{BR}(\bar{B} \to X_s \gamma)_{E_\gamma > 1 \text{ GeV}} = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B} \right)^2 + \ldots \right] + \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B} \right)^2 + \ldots \right]$$
$$+ \mathcal{O} \left[\left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right] + \text{ [Contributions other than the "77" term]}$$

Contributions from Operators containing the charm quark at the leading order in α_s can be expressed as a power series:

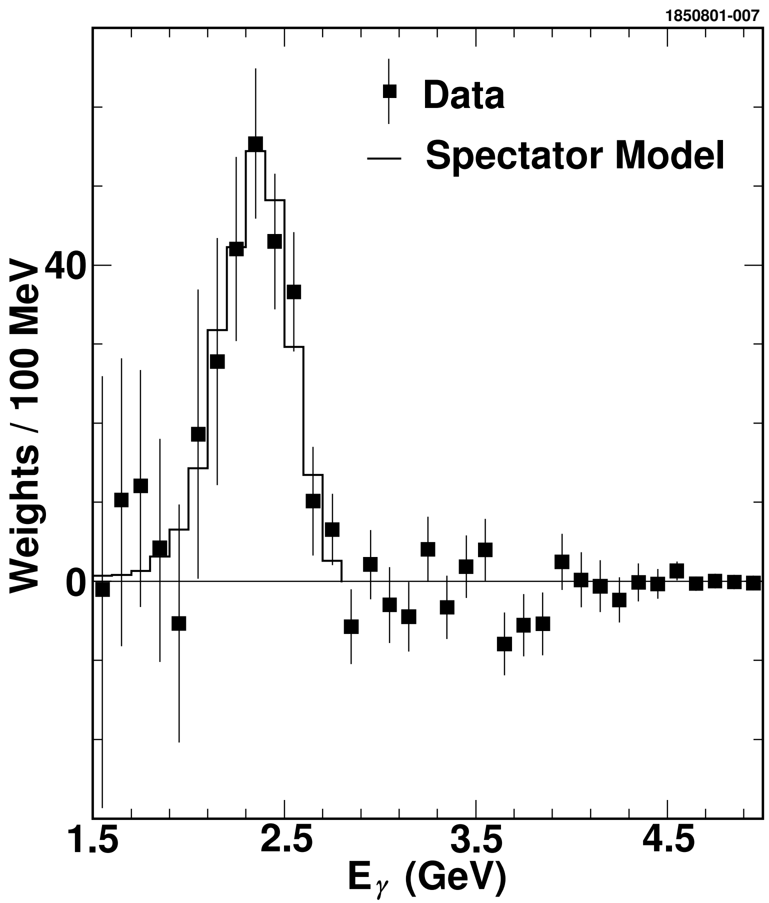
$\langle \bar{B} |$ 

$| \bar{B} \rangle$

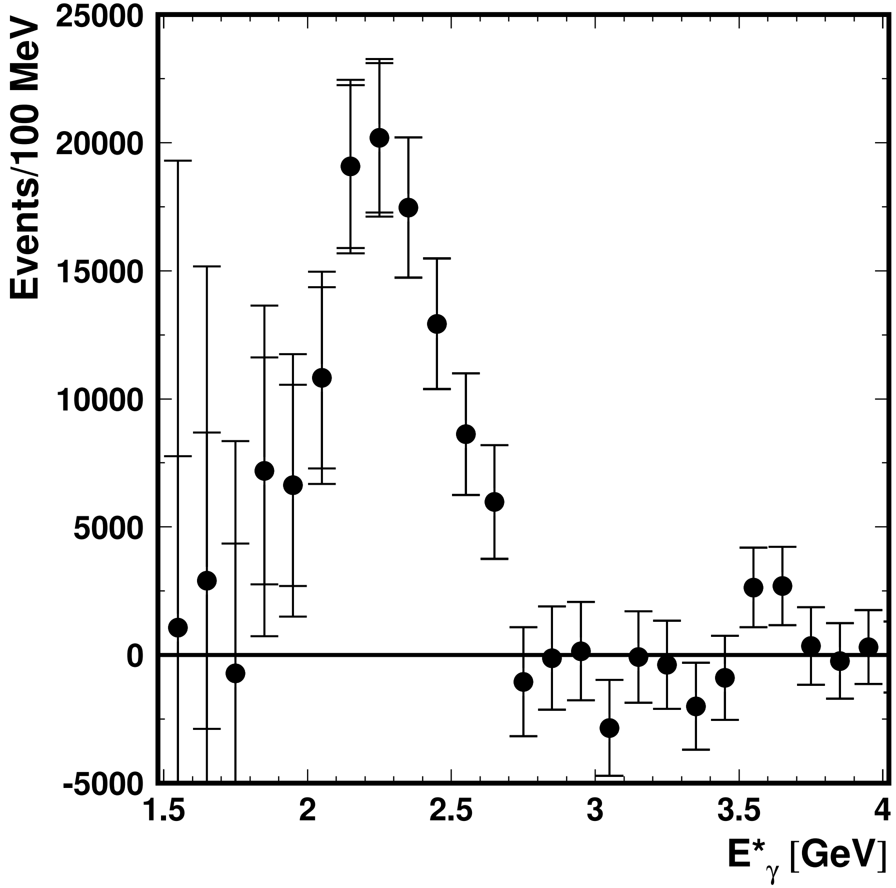
$$= \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda}{m_c^2} \right)^n ,$$

which can be truncated to the leading $n = 0$ term, because the coefficients b_n decrease fast with n. The calculable $n = 0$ term makes BR$[\bar{B} \to X_s \gamma]$ increase by around 3%.

Measurement of $\bar{B} \to X_s \gamma$



CLEO
hep-ex/0108032
PRL 87 (2001) 251807

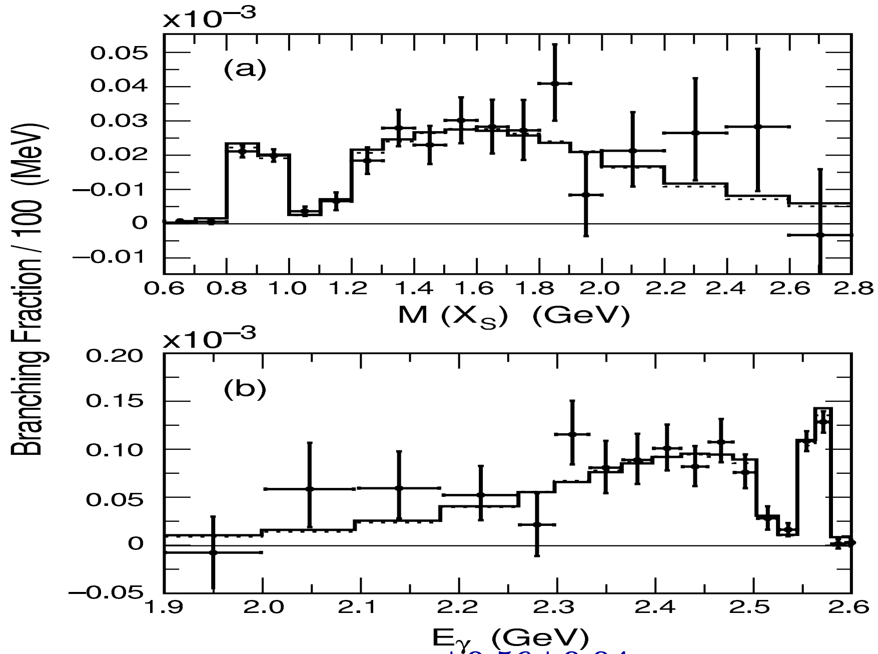


BELLE
hep-ex/0403004

Photon Energy Spectrum from Sum of Exclusive Final States

BABAR Collaboration hep-ex/0508004

- Theory: Shape function [Kagan, Neubert; Neubert et al.]
  Kinetic quark mass scheme: [Benson, Bigi, Uraltsev]



- $\mathcal{B}(B \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} = (3.35 \pm 0.19^{+0.56+0.04}_{-0.41-0.09}) \times 10^{-4}$
- Isospin-asymmetry:

$$\Delta_{0-} = \frac{\Gamma(\bar{B}^0 \to X_{s\bar{d}}\gamma) - \Gamma(B^- \to X_{s\bar{u}}\gamma)}{\Gamma(\bar{B}^0 \to X_{s\bar{d}}\gamma) + \Gamma(B^- \to X_{s\bar{u}}\gamma)} = -0.006 \pm 0.058 \pm 0.009$$

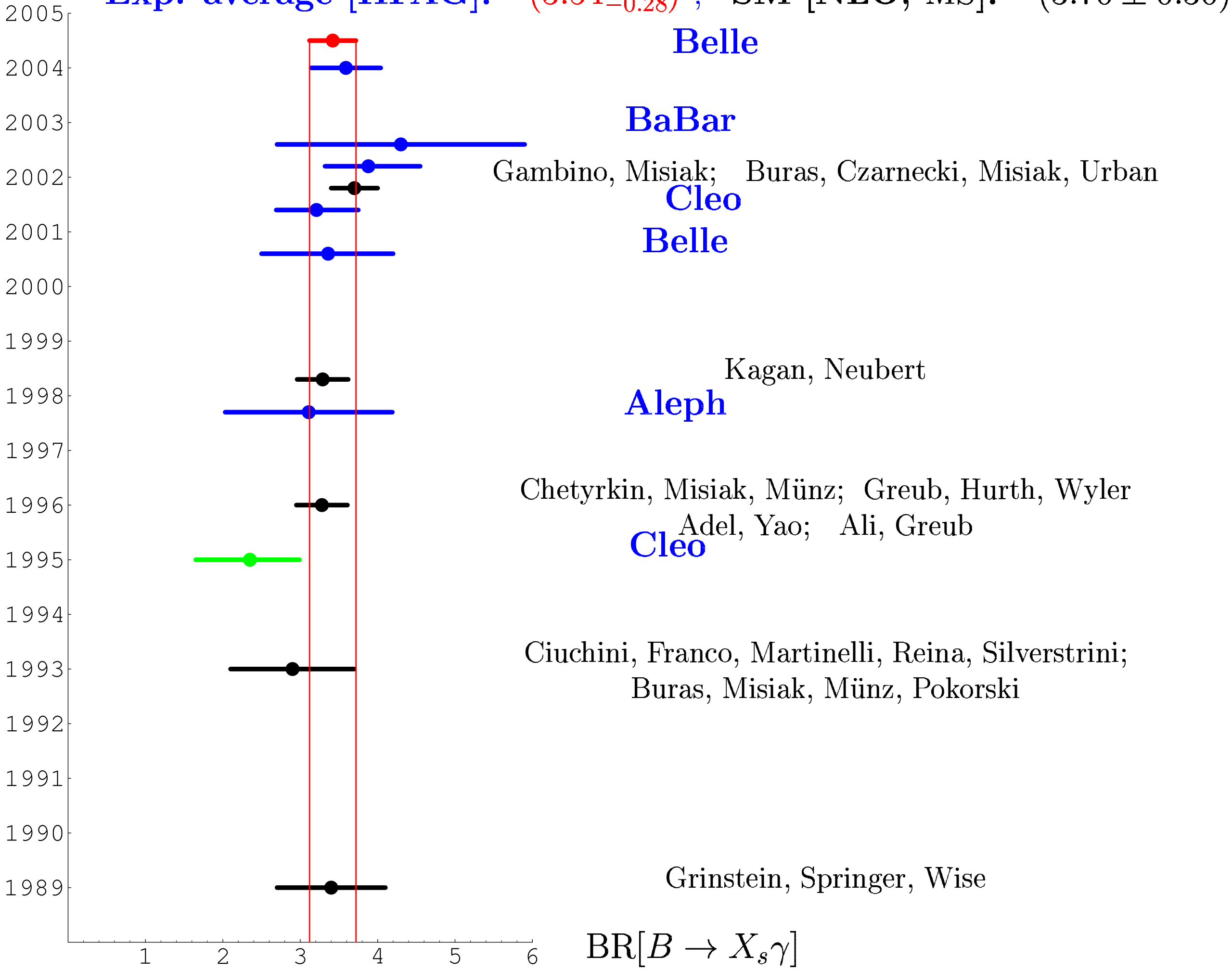
- consistent with SM, where Δ_{0-} power (Λ/m_b) suppressed; typically a few %

Evolution in time

$\mathrm{BR}[\bar{B} \to X_s \gamma]$ **(units: 10^{-4})**

Measurements & the SM calculations

Exp. average [HFAG]: $(3.54^{+0.30}_{-0.28})$; **SM [NLO; $\overline{\mathrm{MS}}$]:** (3.70 ± 0.30)



Determination of V_{ts} from BR $(\bar{B} \to X_s\gamma)$

- Unitarity of the CKM Matrix

$$\sum_{u,c,t} \lambda_i = 0, \quad \text{with} \quad \lambda_i = V_{ib}V_{is}^*$$

- $\lambda_u = V_{ub}V_{us}^* \simeq A\lambda^4(\bar{\rho} - i\bar{\eta}) \simeq O(10^{-2})$
- $\lambda_t = -\lambda_c = -A\lambda^2 + ... = -(41.0 \pm 2.1) \times 10^{-3}$
- Without invoking the CKM unitarity, NLO SM-calculations in the $\overline{\text{MS}}$ scheme and current data $\mathcal{B}(B \to X_s\gamma) = (3.39^{+0.30}_{-0.27}) \times 10^{-4}$ imply the constraint [Misiak, AA]

$$|1.69\lambda_u + 1.60\lambda_c + 0.60\lambda_t| = (0.92 \pm 0.07)|V_{cb}|$$

$$\Longrightarrow \lambda_t = V_{tb}V_{ts}^* = -(46.0 \pm 8.0) \times 10^{-3}$$

- In future, NNLO calculations will lead to a determination of BR$(\bar{B} \to X_s\gamma)$ to an accuracy of 5%
- With improved data, this will determine V_{ts} to an accuracy of about 10%

Fraction $R(E_0)$ of BR $(B \to X_s\gamma)$ above the cut $E_\gamma > E_0$

- In experiments, a lower cut-off on E_γ is required to reduce the background; theory and experiment can be compared for $E_\gamma > E_0$; to quote the full BR, need to evaluate the fraction $R(E_0)$ of the events surviving this cut
- $R(E_0)$ usually calculated using (model-dependent) shape functions [Kagan, Neubert; Benson, Bigi, Uraltsev,...]
- Recently, it has been pointed out [Neubert, hep-ph/0408179] that $R(E_0)$ can be calculated without reference to shape functions using a multi-scale OPE
- Theoretical framework for this calculation is the so-called Soft Collinear Effective Theory (SCET) involving several scales: m_b, $m_b\Delta$, and Δ, with $\Delta = m_b - 2E_0$
- Large logarithms associated with these scales are summed at NLL order; sensitivity to the scale $\Delta \simeq 1.1$ GeV (for $E_0 = 1.8$ GeV) introduces additional uncertainties. Thus,

$$R(E_0) = (92^{+7}_{-10} \pm 1)\% \quad \Longrightarrow \mathcal{B}(B \to X_s\gamma) = (3.38^{+0.31+0.32}_{-0.42-0.30}) \times 10^{-4}$$

- First error is an estimate of the perturbative uncertainties in the multiple scale theory, which in principle can be reduced by improved calculations; second error is due to the input parameters
- The corresponding experimental BR from BELLE is:

$$\mathcal{B}(B \to X_s\gamma) = (3.38 \pm 0.30 \pm 0.29) \times 10^{-4}$$

$B \to (K^*, \rho)\, \gamma$ decay rates in NLO

- For Large $E_V \sim m_B/2$, symmetries in effective theory $\Longrightarrow$ relations among FFs:

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2)$$

- Symmetries in effective theory broken by perturbative QCD

Factorization Ansatz:

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2) + \Phi_B \otimes T_k \otimes \Phi_V$$

Perturbative Corrections:

$$C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + \ldots$$

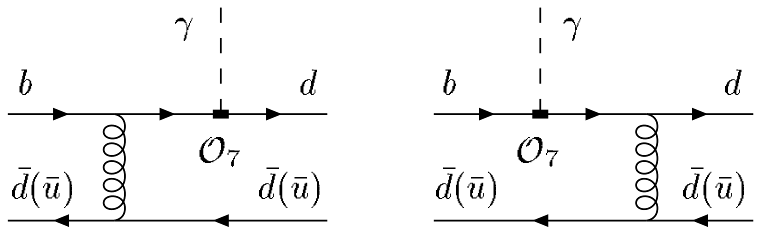
- T_k: Hard Spectator Corrections

$$\Delta\mathcal{M}^{(\mathrm{HSA})} \propto \int_0^1 du \int_0^\infty dl_+\, M^{(B)} M^{(V)}\, T_k$$

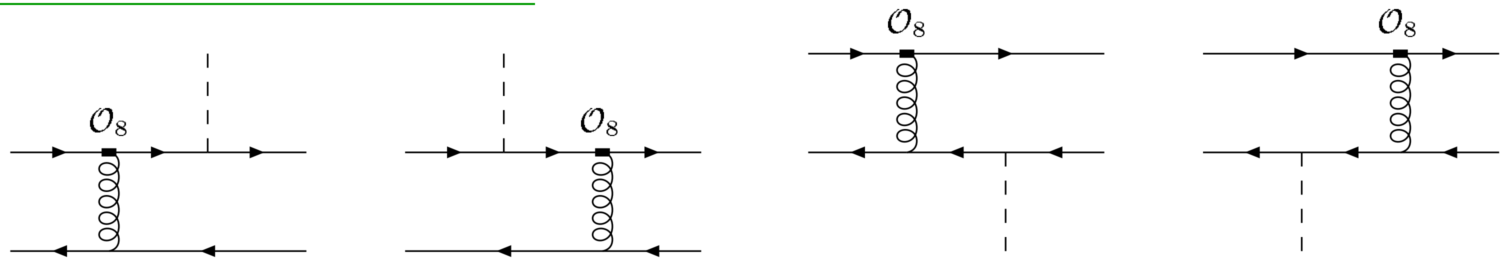
- $M^{(B)}$ and $M^{(V)}$ B-Meson & V-Meson Projection Operators

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

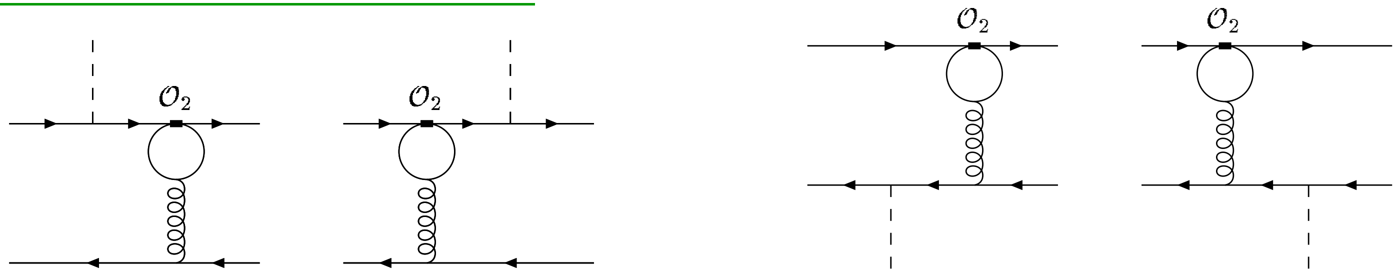
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



NLO Calculation of $\mathcal{B}(B \to K^*\gamma)$

[Parkhomenko, A.A.; Beneke, Feldmann, Seidel; Bosch, Buchalla]

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2$$

- $A^{(1)}(\mu)$ has the decomposition [Parkhomenko, A.A.; hep-ph/0105302]

$$A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)K^*}(\mu_{\rm sp})$$

$$\begin{aligned}
A_{C_7}^{(1)}(\mu) &= \frac{\alpha_s(\mu)}{4\pi} C_7^{(1)\rm eff}(\mu) \\
A_{\rm ver}^{(1)}(\mu) &= \frac{\alpha_s(\mu)}{4\pi} \left\{ \frac{32}{81} \left[13 C_2^{(0)}(\mu) + 27 C_7^{(0)\rm eff}(\mu) - 9\, C_8^{(0)\rm eff}(\mu)\right] \ln\frac{m_b}{\mu} \right. \\
&\quad - \left. \frac{20}{3} C_7^{(0)\rm eff}(\mu) + \frac{4}{27}\left(33 - 2\pi^2 + 6\pi i\right) C_8^{(0)\rm eff}(\mu) + r_2(z)\, C_2^{(0)}(\mu) \right\} \\
A_{\rm sp}^{(1)K^*}(\mu_{\rm sp}) &= \frac{\alpha_s(\mu_{\rm sp})}{4\pi} \frac{2\Delta F_\perp^{(K^*)}(\mu_{\rm sp})}{9\xi_\perp^{(K^*)}} \left\{ 3 C_7^{(0)\rm eff}(\mu_{\rm sp}) \right. \\
&\quad + \left. C_8^{(0)\rm eff}(\mu_{\rm sp}) \left[1 - \frac{6 a_{\perp 1}^{(K^*)}(\mu_{\rm sp})}{\langle \bar{u}^{-1}\rangle_\perp^{(K^*)}(\mu_{\rm sp})}\right] + C_2^{(0)}(\mu_{\rm sp}) \left[1 - \frac{h^{(K^*)}(z,\mu_{\rm sp})}{\langle \bar{u}^{-1}\rangle_\perp^{(K^*)}(\mu_{\rm sp})}\right] \right\}
\end{aligned}$$

- $z = m_c^2/m_b^2$; $\mu_{\rm sp} = \sqrt{\mu \Lambda_H}$; $\Lambda_H = O(\Lambda_{\rm QCD})$

Comparison with data

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 K_{\rm NLO} \left|C_7^{(0)\rm eff}\right|^2$$

$$K_{\rm NLO} = \frac{\left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0)\rm eff}\right|^2} \quad \text{with} \quad 1.5 \leq K \leq 1.7$$

$$\mathcal{B}_{\rm th}(B^0 \to K^{*0}\gamma) \simeq (6.9 \pm 1.1) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

$$\mathcal{B}_{\rm th}(B^\pm \to K^{*\pm}\gamma) \simeq (7.4 \pm 1.2) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

- $T_1^{K^*}(0) = (1 + O(\alpha_s))\xi_\perp^{(K^*)}(0)$
  [Beneke, Feldmann]

Current Experimental Average

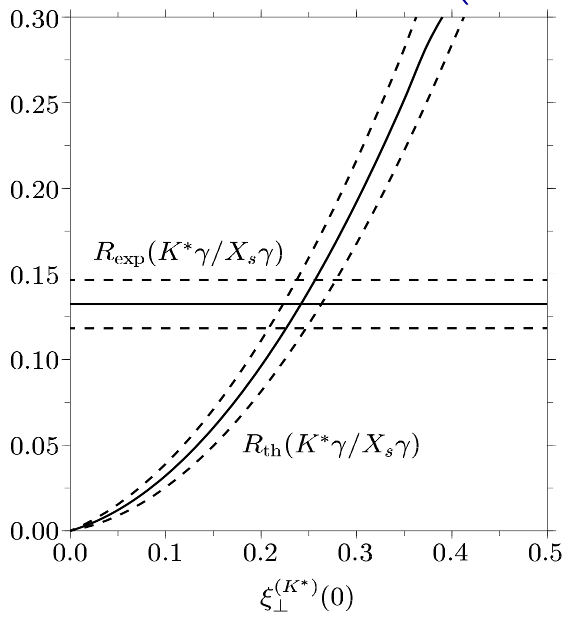
$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.14 \pm 0.26) \times 10^{-5}$

$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (3.98 \pm 0.35) \times 10^{-5}$

- Using the ratio

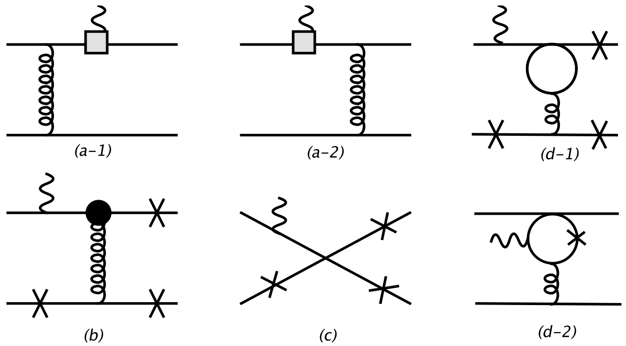
$R_{\rm exp}(K^*\gamma/X_s\gamma) = 0.117 \pm 0.012$

$\Longrightarrow \quad T_1^{K^*}(0) = 0.27 \pm 0.02$



$B \to K^*\gamma$ in PQCD

[Keum, Matsumori, Sanda]



$$
\begin{aligned}
Br(B^0 \to K^{*0}\gamma) &= (4.9 \pm 2.5) \times 10^{-5} \\
Br(B^\pm \to K^{*\pm}\gamma) &= (5.0 \pm 2.5) \times 10^{-5}
\end{aligned}
$$

$\Longrightarrow$ Form factor: $T_1^{K^*}(0) = 0.23 \pm 0.06$

in agreement with QCDF-based estimates of the same and data

- Isospin Symmetry Breaking :

$$
\Delta_{0-} = \frac{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) - Br(B^- \to K^{*-}\gamma)}{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) + Br(B^- \to K^{*-}\gamma)} = (3.0 \pm 0.9)\%
$$

[Cf: $\Delta_{0-} = (8 \pm 4)\%$ [Kagan, Neubert (QCDF)]]

- $\Delta_{0-}(K^*\gamma)^{exp} = (3.9 \pm 4.8)\%$

$B \to \rho\gamma$ decay in LO

- Effective Weak Hamiltonian

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{ud}^* (C_1 O_1 + C_2 O_2) - V_{tb} V_{td}^* C_7^{\rm eff} O_7 + ... \right]$$

- **Penguin amplitude dominated by O_7: $\mathcal{M}_{\rm P}(B \to \rho\gamma)$**

$$-\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* C_7 \frac{e m_b}{4\pi^2} \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta - i \left[g^{\mu\nu} (q.p) - p^\mu q^\nu \right] \right) T_1^{(\rho)}(0)$$

- **Annihilation amplitude dominated by O_1 and O_2: $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$**
- In the factorization approximation:

$$\langle \rho\gamma | O_2 | B \rangle = \langle \rho | \bar{d} \Gamma_\mu u | 0 \rangle \langle \gamma | \bar{u} \Gamma^\mu b | B \rangle + \langle \rho\gamma | \bar{d} \Gamma_\mu u | 0 \rangle \langle 0 | \bar{u} \Gamma^\mu b | B \rangle$$

- This allows to calculate $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$ in terms of the vector and axial vector form factors and hadronic quantities f_B, f_ρ:

$$e \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 m_\rho \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F_A^{(\rho);{\rm pv}} - i \left[g^{\mu\nu} (q.p) - p^\mu q^\nu \right] F_A^{(\rho);{\rm pc}} \right)$$

- The $B \to \rho\gamma$ amplitude requires 3 FFs: $T_1^{(\rho)}$, $F_A^{(\rho);{\rm pv}}$ and $F_A^{(\rho);{\rm pc}}$

- Light-cone QCD sum rules [Braun,A.A.; Khodjamirian, Stoll, Wyler]

$$\frac{f_B m_B^2}{m_b f_\rho} F_A^{(\rho)} \exp^{-(m_B^2 - m_b^2)/t} = \int_0^1 \frac{du}{u} \exp^{-(\bar{u}/u) m_b^2/t} \theta(s_0 - m_b^2/u) \left[e_u \langle \bar{\psi}\psi \rangle \chi \phi_\gamma(u) + \ldots \right]$$

$$\frac{f_B m_B^2}{m_b f_\rho} T_1^{(\rho)} \exp^{-(m_B^2 - m_b^2)/t} = \int_0^1 \frac{du}{u} \exp^{-(\bar{u}/u) m_b^2/t} \theta(s_0 - m_b^2/u) \left[m_b \phi_\perp^\rho(u) + \ldots \right]$$

  - $\bar{u} = 1 - u$, t is the Borel parameter, s_0 is the continuum threshold, χ is the magnetic susceptibility $\langle \bar{\psi} \sigma_{\alpha\beta} \psi \rangle_F = e_\psi \langle \bar{\psi}\psi \rangle F_{\alpha\beta}$, and only dominant terms shown

- As the photon and transverse-rho meson wave functions are rather close, one has $F_A^{(\rho);\mathrm{pv}}(0) \simeq F_A^{(\rho);\mathrm{pc}}(0) \equiv F_A^{(\rho)}(0)$ [See, also Byer, Melikhov, Stech]

- In LO, the ratio A/P in $B^\pm \to \rho^\pm \gamma$ estimated as [See, also Grinstein, Pirjol]

$$\epsilon_{\mathrm{A}}(\rho^\pm \gamma) = \frac{4\pi^2 m_\rho a_1}{m_b C_7^{eff}} \frac{F_A^{(\rho)}(0)}{T_1^{(\rho)}} = 0.30 \pm 0.07$$

Annihilation amplitude $\mathcal{M}_{\mathrm{A}}(B^0 \to \rho^0 \gamma)$

- Suppressed due to the electric charges ($Q_d/Q_u = -1/2$) and colour factors (BSW Parameters: $a_2/a_1 \simeq 0.25$)

$$\Longrightarrow \epsilon_{\mathrm{A}}(\rho^0 \gamma) \simeq 0.05$$

- $\epsilon_{\mathrm{A}}(\rho^0 \gamma) \ll \epsilon_{\mathrm{A}}(\rho^\pm \gamma)$ is the dominant source of isospin violation in $B \to \rho\gamma$

$B \to \rho\gamma$ decay in NLO

- Including the $O(\alpha_s)$ vertex and hard spectator correction for the penguin amplitude in the QCD-Factorization approach, and the lowest order result for the annihilation amplitude, one has:

$$
\begin{aligned}
&\Gamma_{\rm th}(B^\pm \to \rho^\pm\gamma) \\
= \;& \frac{G_F^2 \alpha |V_{tb}V_{td}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left(1 - \frac{m_\rho^2}{M^2}\right)^3 \left[\xi_\perp^{(\rho)}(0)\right]^2 \\
\times \;& \Big\{ (C_7^{(0)\rm eff} + A_R^{(1)t})^2 + (F_1^2 + F_2^2)\,(A_R^u + L_R^u)^2 \\
+ \;& 2F_1\,[C_7^{(0)\rm eff}(A_R^u + L_R^u) + A_R^{(1)t} L_R^u] \pm 2F_2\,[C_7^{(0)\rm eff} A_I^u - A_I^{(1)t} L_R^u] \Big\}
\end{aligned}
$$

where

$$
\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = -\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| e^{i\alpha} \equiv F_1 + iF_2; \quad L_{\rm R}^u = \epsilon_A C_7^{(0)\rm eff}
$$

$$
A^{(1)t}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)\rho}(\mu_{\rm sp})
$$

- $A_{C_7}^{(1)}(\mu)$ and $A_{\rm ver}^{(1)}(\mu)$ already encountered in $B \to K^*\gamma$; $A_{\rm sp}^{(1)\rho}(\mu_{\rm sp})$ and $A^u(\mu)$ specific to $B \to \rho\gamma$ decays [See, Parkhomenko, A.A., hep-ph/0105302]
- Very recently, $O(\alpha_s)$ contribution to the annihilation amplitudes in $B \to \rho\gamma$ calculated, leading to small changes in the branching ratios [Pilipp; Chamonix Talk]

$B \to (\rho,\ \omega)\gamma$ decay rates

[Parkhomenko, A.A.; Bosch, Buchalla; Lunghi, Parkhomenko, AA; Beneke, Feldmann, Seidel]

$$R(\rho\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \rho\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\rho^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3}\,\zeta^2\ [1 + \Delta R(\rho/K^*)]$$

$$R(\omega\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \omega\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = 1/2 \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\omega^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3}\,\zeta^2\ [1 + \Delta R(\omega/K^*)]$$

- $S_\rho = 1$ for $B^\pm \to \rho^\pm\gamma$; $= 1/2$ for $B^0 \to \rho^0\gamma$
- $\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.85 \pm 0.10$; $T_1^\omega(0) = T_1^{(\rho)}(0)$ [QCD − SRs, Lattice]
- $\Delta R(\rho^\pm/K^{*\pm}) = 0.12 \pm 0.10$
- $\Delta R(\rho^0/K^{*0}) \simeq \Delta R(\omega/K^{*0}) = 0.1 \pm 0.07$

Theoretical Branching Ratios [Lunghi, Parkhomenko, AA]

- $R(\rho^\pm/K^{*\pm}) = (3.3 \pm 1.0) \times 10^{-2}$
- $R(\rho^0/K^{*0}) \simeq R(\omega/K^{*0}) = (1.6 \pm 0.5) \times 10^{-2}$
- $\mathrm{BR}(\mathrm{B}^\pm \to \rho^\pm\gamma) = (1.35 \pm 0.4) \times 10^{-6}$
- $\mathrm{BR}(\mathrm{B}^0 \to \rho^0\gamma) \simeq \mathrm{BR}(\mathrm{B}^0 \to \omega\gamma) = (0.65 \pm 0.2) \times 10^{-6}$

Experiment vs. SM $(b \to d\gamma)$

SM Estimates [Lunghi, Parkhomenko, AA]

$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}$$
$$= (1.38 \pm 0.42) \times 10^{-6}$$

$$R[(\rho,\omega)/K^*] \equiv \frac{\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma]}{\bar{\mathcal{B}}[B \to K^*\gamma]} = 0.033 \pm 0.010$$

BELLE

$$\bar{\mathcal{B}}_{\text{exp}}[B \to (\rho,\omega)\,\gamma] = (1.34^{+0.34}_{-0.31}\ (\text{stat})^{+0.14}_{-0.10}\ (\text{syst})) \times 10^{-6}$$

$$R[(\rho,\omega)/K^*] = 0.032 \pm 0.008\ (\text{stat})^{+0.003}_{-0.002}\ (\text{syst})$$

$$|V_{td}/V_{ts}| = 0.200^{+0.026}_{-0.025}(\text{exp})\ ^{+0.038}_{-0.029}\ (\text{theo})$$

BABAR

$$\bar{\mathcal{B}}_{\text{exp}}[B \to (\rho,\omega)\,\gamma] < 1.2 \times 10^{-6}\ \ (90\%\ \text{CL})$$

$$R[(\rho,\omega)/K^*] < 0.029 \implies |V_{td}/V_{ts}| < 0.19$$

D. Mohapatra (BELLE)[EPS 2005]

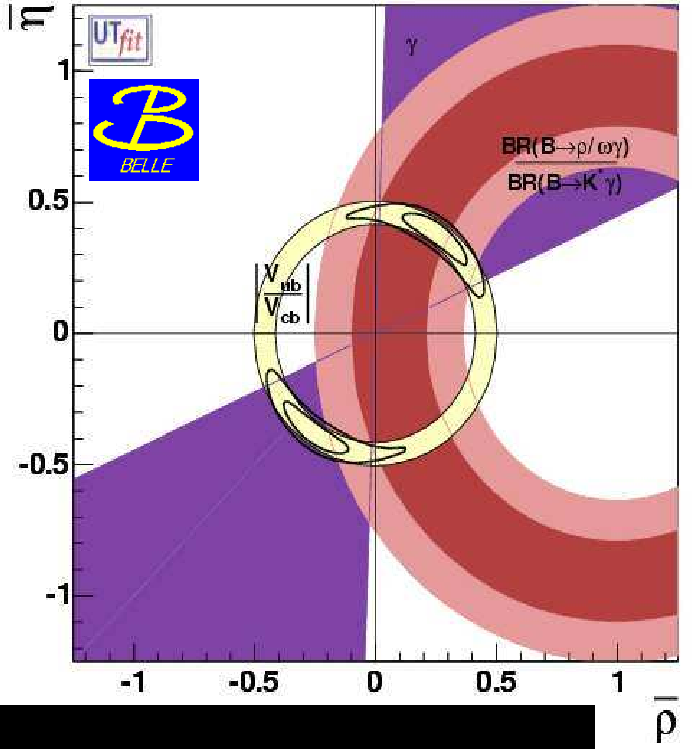
Extraction of $|V_{td}/V_{ts}|$

$$\frac{B(\overline{B}\to(\rho,\omega)\gamma)}{B(B\to K^*\gamma)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{1-M_\rho^2/M_B^2}{1-M_{K^*}^2/M_B^2}\right)\zeta^2[1+\Delta R]$$

Form factor ratio $\zeta = 0.85 \pm 0.10$

SU(3)-breaking effect $\Delta R = 0.1 \pm 0.1$

$$\frac{B(B\to(\rho,\omega)\gamma)}{B(B\to K^*\gamma)} = 0.032 \pm 0.008^{+0.003}_{-0.002}$$



$$0.143 < \left|\frac{V_{td}}{V_{ts}}\right| < 0.260$$

(95 % C.L. interval)

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.200^{+0.026}_{-0.025}(\text{expt.})^{+0.038}_{-0.029}(\text{theo.})$$

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Isospin violation in $B \to \rho\gamma$ decays

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right], \qquad \Delta^{\pm 0} \equiv \frac{\Gamma(B^\pm \to \rho^\pm\gamma)}{2\Gamma(B^0(\bar{B}^0) \to \rho^0\gamma)} - 1$$

$$\Delta_{\rm LO} = 2\epsilon_A\left[F_1 + \frac{\epsilon_A}{2}\left(F_1^2 + F_2^2\right)\right] = 2\epsilon_A F\cos\alpha + O(\epsilon_A^2)$$

$$\Delta_{\rm NLO} \simeq \Delta_{\rm LO} - \frac{2\epsilon_A}{C_7^{(0)\rm eff}} F\cos\alpha\left[A_R^{(1)t} + A_R^u F\cos 2\alpha\right] + O(\epsilon_A^2)$$

$$F_1 = F\cos\alpha; \quad F_2 = F\sin\alpha; \quad F = \frac{R_b}{R_t} \simeq 0.5$$

$$\Delta = (1.1 \pm 3.9)\% \ \text{ for } \ \alpha = (92 \pm 11)^\circ$$

$$\Delta^{(\rho/\omega)} \equiv \tfrac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right]$$

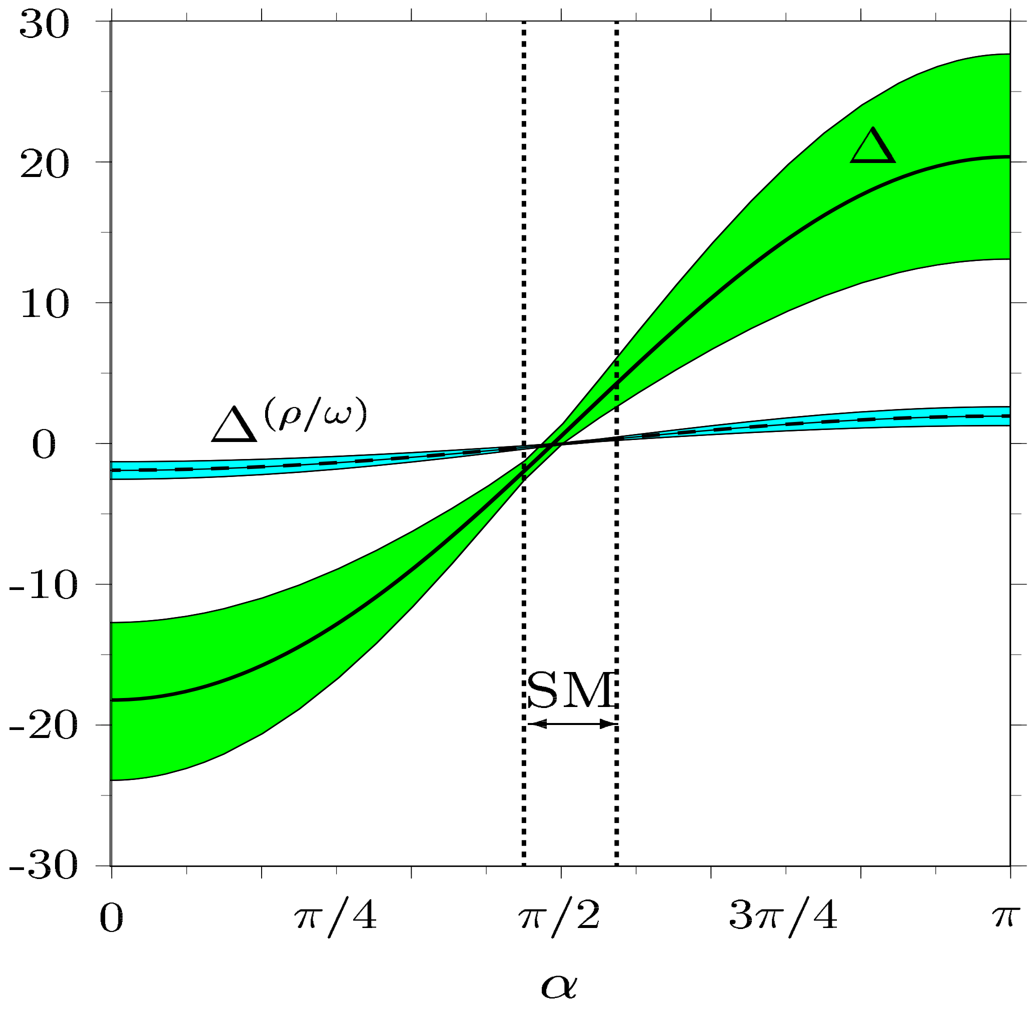
$$\Delta_B^{(\rho/\omega)} \equiv \frac{(M_B^2 - m_\omega^2)^3\mathcal{B}(B^0 \to \rho\gamma) - (M_B^2 - m_\rho^2)^3\mathcal{B}(B^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3\mathcal{B}(B^0 \to \rho\gamma) + (M_B^2 - m_\rho^2)^3\mathcal{B}(B^0 \to \omega\gamma)}$$

with $\Delta_{\bar{B}}^{(\rho/\omega)} = \Delta_B^{(\rho/\omega)}(B^0 \to \bar{B}^0)$

$$\Delta_B^{(\rho/\omega)} = (0.3 \pm 3.9) \times 10^{-3} \ \text{ for } \ \alpha = (92 \pm 11)^\circ$$

Isospin-violating ratio Δ in $B \to \rho\gamma$ decays

[AA, Lunghi, Parkhomenko; hep-ph/0405075]



CP Asymmetry in $B \to (\rho, \omega)\gamma$ decays

Direct CP Asymmetry in $B \to (\rho, \omega)\gamma$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\rho^\pm\gamma) \equiv \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)},$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma) \equiv \frac{\mathcal{B}(\bar{B}_d^0 \to \rho^0\gamma) - \mathcal{B}(B_d^0 \to \rho^0\gamma)}{\mathcal{B}(\bar{B}_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \rho^0\gamma)}$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\omega\gamma) \equiv \frac{\mathcal{B}(\bar{B}_d^0 \to \omega\gamma) - \mathcal{B}(B_d^0 \to \omega\gamma)}{\mathcal{B}(\bar{B}_d^0 \to \omega\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)}$$

$$\mathcal{A}_{\rm CP}(\rho/\omega\gamma) = \frac{2F\sin\alpha(A_I^u - \epsilon_A A_I^{(1)t})}{C_7^{(0)\rm eff}(1+\Delta_{\rm LO})}$$

Mixing-induced CP Asymmetery in $B^0 \to (\rho, \omega)\gamma)$

$$a_{\rm CP}^{\rho\gamma}(t) = -C_{\rho\gamma}\cos(\Delta M_d t) + S_{\rho\gamma}\sin(\Delta M_d t)$$

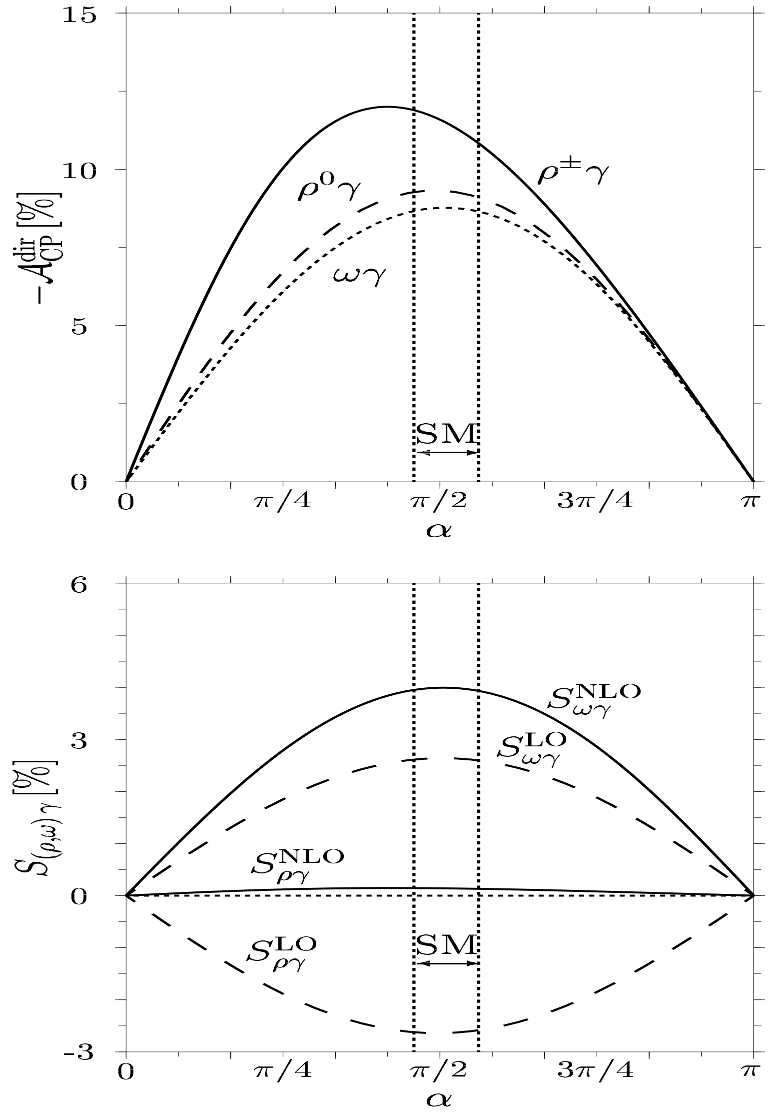
$$\lambda_{\rho\gamma} \equiv \frac{q}{p}\,\frac{A(\bar{B}_d^0 \to \rho^0\gamma)}{A(B_d^0 \to \rho^0\gamma)} = \frac{C_7^{(0)\rm eff} + A^{(1)t} - [C_7^{(0)\rm eff}\varepsilon_A^{(0)} + A^u]\,F\,\mathrm{e}^{+i\alpha}}{C_7^{(0)\rm eff} + A^{(1)t} - [C_7^{(0)\rm eff}\varepsilon_A^{(0)} + A^u]\,F\,\mathrm{e}^{-i\alpha}}$$

where $p/q \simeq \exp(2i\beta)$ and $F = R_b/R_t$

$$C_{\rho\gamma} = -\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma) = \frac{1-|\lambda_{\rho\gamma}|^2}{1+|\lambda_{\rho\gamma}|^2}, \qquad S_{\rho\gamma} = \frac{2\,{\rm Im}(\lambda_{\rho\gamma})}{1+|\lambda_{\rho\gamma}|^2}$$

CP-violating Asymmetries in $B \to (\rho, \omega)\gamma$ decays

[AA, Lunghi, Parkhomenko; hep-ph/0405075]



$\bar{B} \to X_s l^+ l^-$

- The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

- Coefficients of the two additional operators

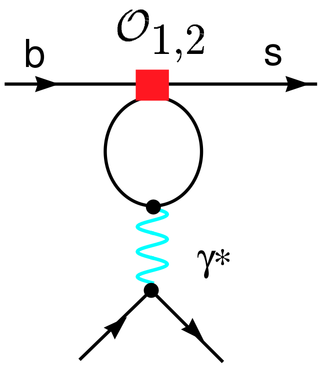
$$O_i = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), \qquad i = 9, \mathbf{10}$$

have the following perturbative expansion:

$$\begin{aligned} C_9(\mu) &= \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots \\ C_{10} &= C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots \end{aligned}$$

- After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electro-weak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$



$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Longrightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$

On the other hand: $\quad C_9^{(0)}(m_b) \simeq 2.2$

NNLO Calculations of BR($\bar{B} \to X_s \ell^+ \ell^-$)

- Two-loop matching, three-loop mixing and two-loop matrix elements have been completed
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements:
    [Asatryan, Asatrian, Greub, Walker;
    Asatrian, Bieri, Greub, Hovhannissyan;
    Ghinculov, Hurth, Isidori, Yao;
    Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in $B \to X_s \ell^+ \ell^-$ decays
  - $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi,Hiller; Buchalla, Isidori]
  - $1/m_c$ corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of $B \to X_s \ell^+ \ell^-$ decays [AA, Greub, Hiller, Lunghi]
  - $\mathrm{BR}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\mathrm{BR}(\bar{B} \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

Electroweak Penguins $b \to s\ell^+\ell^-$

- $B \to X_s\ell^+\ell^-$ decay rate

$$\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.46^{+0.98}_{-0.96}) \times 10^{-6} \quad [\mathrm{HFAG'05}]$$

$$SM : (4.2 \pm 0.7) \times 10^{-6} \quad [\mathrm{AGHL'01}]; \quad (4.6 \pm 0.8) \times 10^{-6} \quad [\mathrm{GHIY'04}]$$

- Differential distributions in $B \to X_s\ell^+\ell^-$

• $M(X_s)$-distribution: tests $s \to X_s$ fragmentation model; current FMs provide reasonable fit to data

• $q^2 = M^2_{\ell^+\ell^-}$-distribution away from the $J/\psi, \psi', \ldots$ resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but the precision is not better than 25%

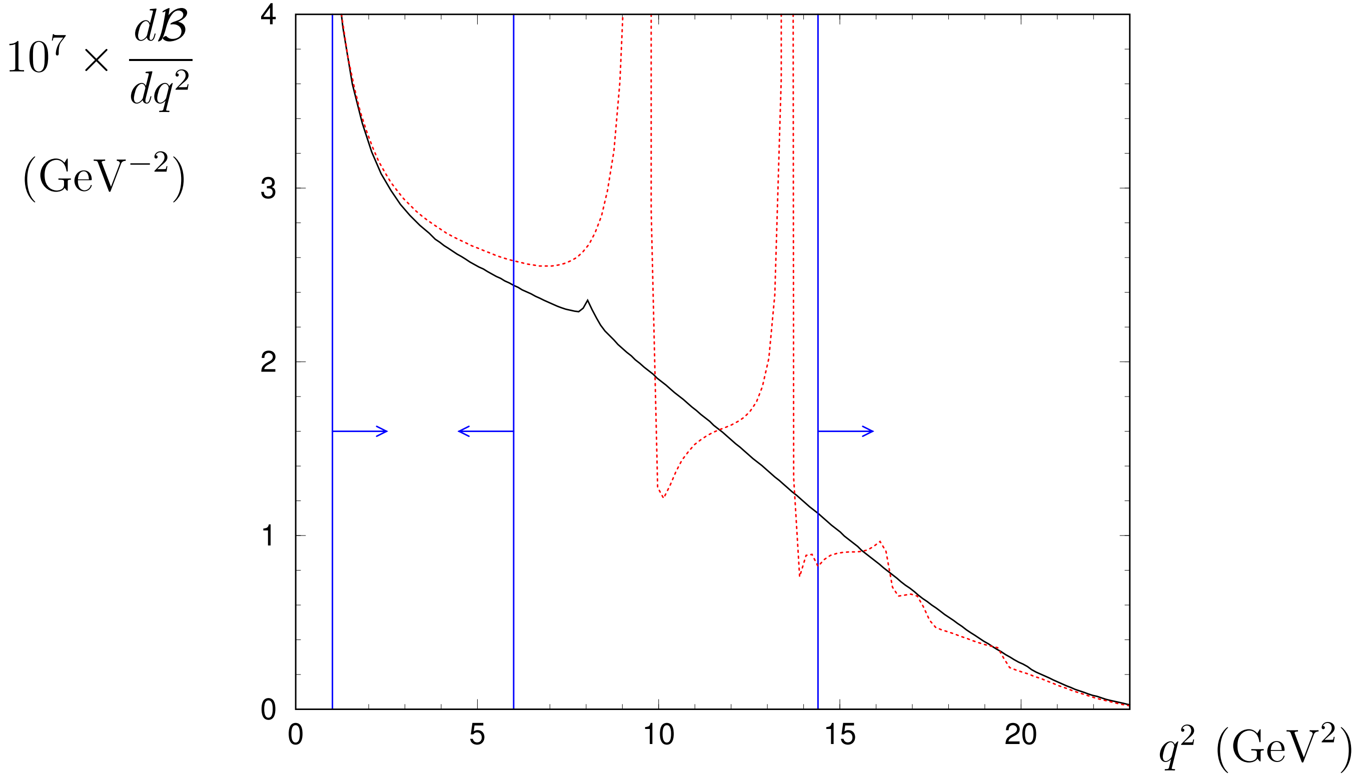
• Forward-Backward Asymmetry (FBA) is likewise sensitive to the SM and BSM effects, in particular encoded in the Wilson coefficients C_7, C_9 and C_{10}

$$A_{\mathrm{FB}}(\hat{s} \sim C_{10}(2C_7 + C_9(\hat{s})\hat{s}); \quad \hat{s} = q^2/M_B^2$$

- $A_{\mathrm{FB}}(\hat{s})$ not yet measured; possible only in experiments at B factories

Dilepton invariant mass distribution in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]

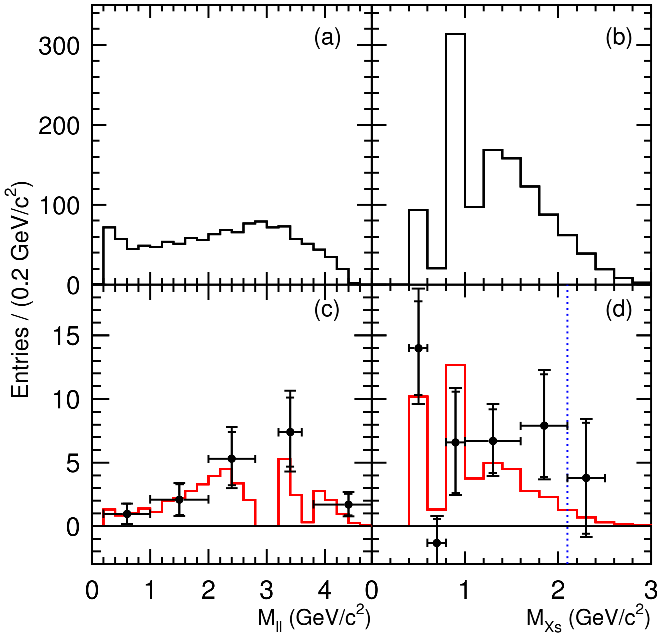


- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $\quad q^2 \in [1, 6]\ \mathrm{GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $\quad q^2 > 14\ \mathrm{GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\mathbf{BR}(\bar{B} \to X_s \mu^+ \mu^-)$; $\quad q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$, in agreement with the earlier NNLO analysis [AA, Greub, Hiller, Lunghi 2001; Bobeth, Gambino, Gorbahn, Haisch, 2003]

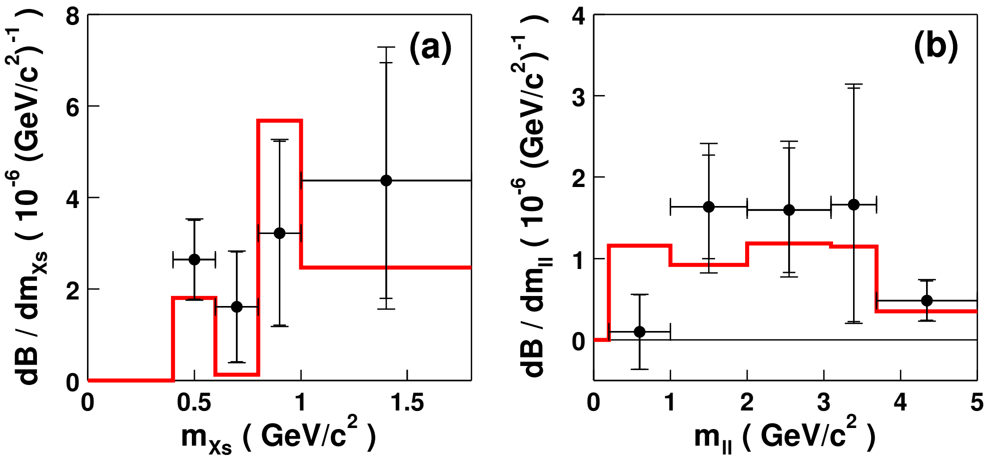
Decay distributions in $\bar{B} \to X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE]



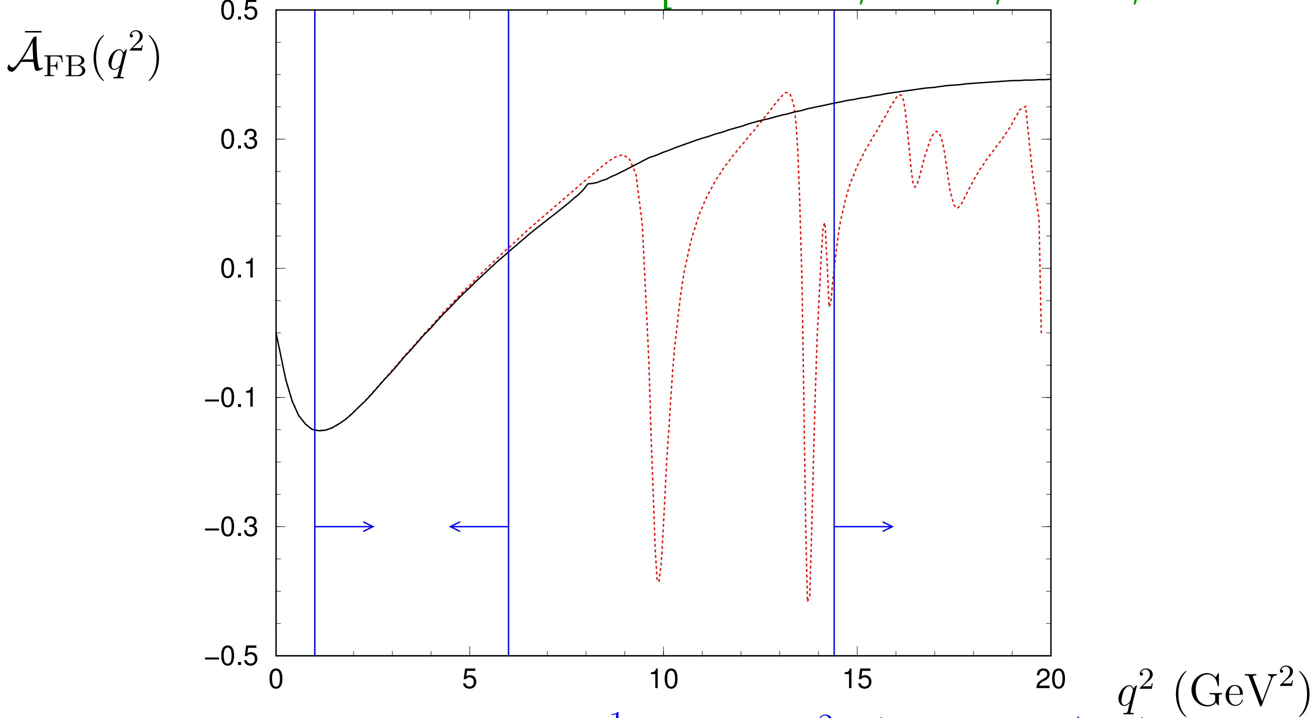
[BABAR]



- In agreement with the NNLO SM calculations

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \; d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$q_0^2 = (3.90 \pm 0.25)$ GeV^2 [Ghinculov, Hurth, Isidori, Yao 2004]

$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b})$ GeV^2 [Bobeth, Gambino, Gorbahn, Haisch 2003]

Electroweak Penguins $b \to s\ell^+\ell^-$

- $B \to (K, K^*)\ell^+\ell^-$ decay rates

  • Decay rates and distributions depend on the form factors; estimates given below based on Light-cone QCD Sum Rules [Ball, Hiller, Handoko,AA]; Several competing estimates available in the literature [Zhong et al; Melnikov et al.;...]

  $\mathcal{B}(B \to K\ell^+\ell^-) = (0.45 \pm 0.05) \times 10^{-6}$ [HFAG'05]; $(0.35 \pm 0.12) \times 10^{-6}$ [SM]

  $\mathcal{B}(B \to K^* e^+ e^-) = (1.26 \pm 0.28) \times 10^{-6}$ [HFAG'05]; $(1.6 \pm 0.5) \times 10^{-6}$ [SM]

  $\mathcal{B}(B \to K^* \mu^+ \mu^-) = (1.45 \pm 0.23) \times 10^{-6}$ [HFAG'05]; $(1.2 \pm 0.5) \times 10^{-6}$[SM]

- Differential distributions in $B \to (K, K^*)\ell^+\ell^-$

  • $q^2 = M^2_{\ell^+\ell^-}$-distribution away from the $J/\psi, \psi', \ldots$ resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but theoretical precision is not better than 35% due to FF dependence

- The ratio $\mathcal{B}(B \to K^* \mu^+ \mu^-)/\mathcal{B}(B \to K^* e^+ e^-)$ sensitive to SUSY effects in the large-$\tan\beta$ region due to Higgs effects

  • $A_{\rm FB}(\hat{s})[B \to K\ell^+\ell^-] \simeq 0$ in the SM and most BSM extensions; in agreement with data which is used as a control sample to measure $A_{\rm FB}(\hat{s})[B \to K^*\ell^+\ell^-]$

  • $A_{\rm FB}(\hat{s})$ in $B \to K^*\ell^+\ell^-$ qualitatively similar to $A_{\rm FB}(\hat{s})$ in $B \to X_s\ell^+\ell^-$, except for FF complication; First measurements from BELLE at hand, appear SM-like; Super-B and LHC-B will measure $A_{\rm FB}(\hat{s})$ precisely

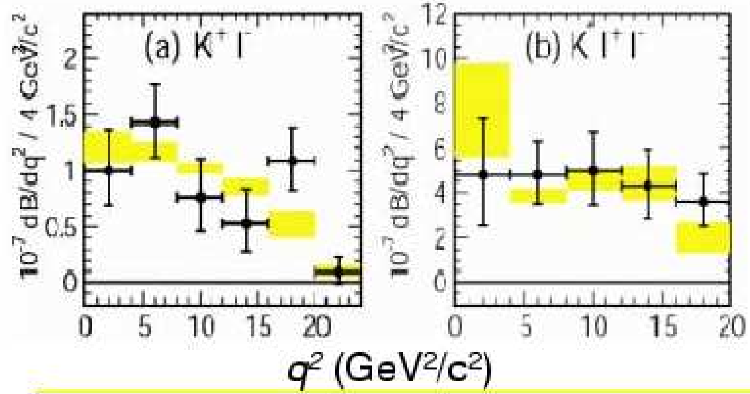
Sakai (FPCP 2004)

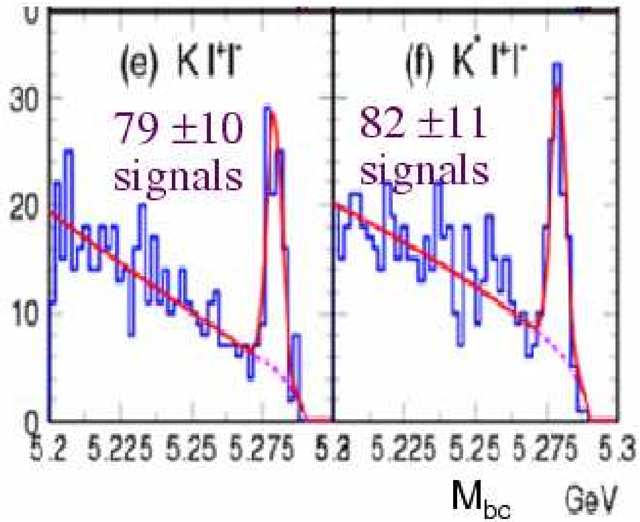
$B \to K^{(*)} l^+ l^-$ [Belle-conf-0415]

LP03: $B \to X_s ll$, $K^{(*)}ll$: Belle/BaBar
Br, A_{CP} ~SM

BELLE **275M $B\bar{B}$** update **>10σ signals**

$\mathcal{B}$(Kll)= $(5.50 \pm {}^{0.75}_{0.70} \pm 0.27 \pm 0.02)$
$\mathcal{B}$(K*ll)= $(16.5 \pm {}^{2.3}_{2.2} \pm 0.9 \pm 0.4)$
x 10^{-7}





ICHEP 2004, Beijing 26

Comparison of $\mathcal{B}(B \to (K, K^*)\ell^+\ell^-$ with SM-1

$B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$

Belle branching fractions (253 fb^{-1})

– $K\ell^+\ell^-$: $(5.50^{+0.75}_{-0.70} \pm 0.27 \pm 0.02) \times 10^{-7}$

– $K^*\ell^+\ell^-$: $(16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7}$

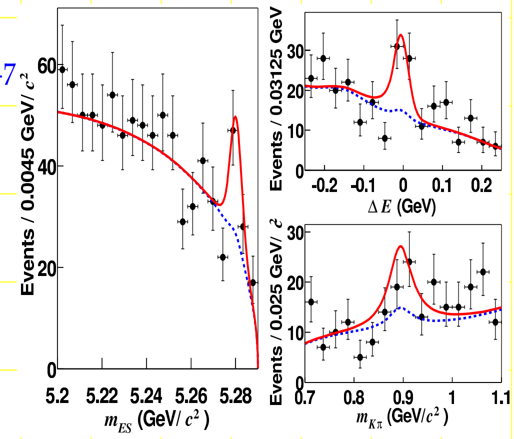
New BaBar results (208 fb^{-1})

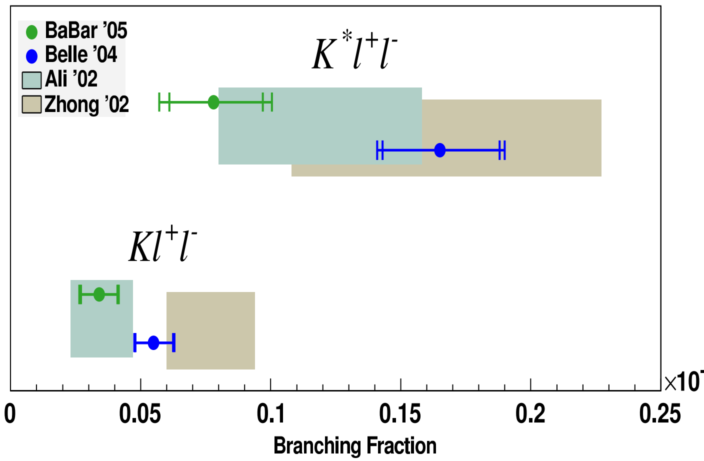
– $K\ell^+\ell^-$: $(3.4 \pm 0.7 \pm 0.3) \times 10^{-7}$

– $K^*\ell^+\ell^-$: $(7.8^{+1.9}_{-1.7} \pm 1.2) \times 10^{-7}$

$A_{CP}(B^+ \to K^+\ell^+\ell^-) = -0.08 \pm 0.22 \pm 0.11$

$A_{CP}(B \to K^*\ell^+\ell^-) = +0.03 \pm 0.23 \pm 0.12$





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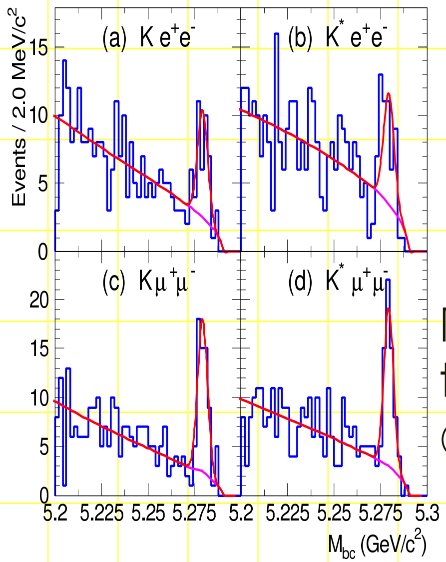
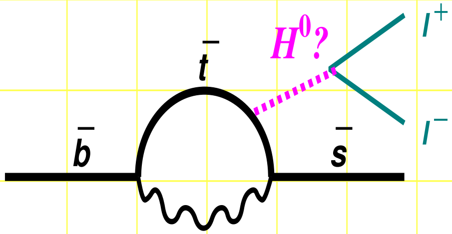
Comparison of $\mathcal{B}(B \to (K, K^*)\ell^+\ell^-$ with SM-2

$B \to K^{(*)}e^+e^-$ vs $B \to K^{(*)}\mu^+\mu^-$





Ratio of $K^{(*)}\mu^+\mu^-$ to $K^{(*)}e^+e^-$ is sensitive to neutral SUSY Higgs if $\tan\beta$ is large

($O(1)$ enhancement if $\tan\beta \sim 30$)

Belle ratios:

$\mathcal{B}(B \to K\mu^+\mu^-)/\mathcal{B}(B \to Ke^+e^-) = 1.38\,^{+0.39}_{-0.41}\,^{+0.06}_{-0.07}$ (1.00 in SM)

$\mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-) = 0.98\,^{+0.30}_{-0.31} \pm 0.08$ ($\sim$ 0.75 in SM)

Babar ratios:

$\mathcal{B}(B \to K\mu^+\mu^-)/\mathcal{B}(B \to Ke^+e^-) = 1.06 \pm 0.48 \pm 0.05$ (1.00 in SM)

$\mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-) = 0.93 \pm 0.46 \pm 0.06$ ($\sim$ 0.75 in SM)

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Forward-Backward Asymmetry in $B \to K^*\ell^+\ell^-$

$$\frac{dA_{FB}}{d\hat{s}} = -\int_0^{\hat{u}(\hat{s})} d\hat{u}\frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} d\hat{u}\frac{d\Gamma}{d\hat{u}d\hat{s}}$$

$$\sim C_{10}[\mathrm{Re}(C_9^{eff})VA_1 + \frac{\hat{m}_b}{\hat{s}}C_7^{eff}(VT_2(1-\hat{m}_V) + A_1T_1(1+\hat{m}_V))]$$

- $\boldsymbol{T_1, T_2, V, A_1}$ form factors
  - Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM $(\hat{s}_0)$ below $m^2_{J/\psi}$

Position of the $A_{FB}(\hat{s})$ zero $(\hat{s}_0)$ in $B \to K^*\ell^+\ell^-$

$$\mathrm{Re}(\mathrm{C_9^{eff}}(\hat{\mathrm{s}}_0)) = -\frac{\hat{\mathrm{m}}_\mathrm{b}}{\hat{\mathrm{s}}_0}\mathrm{C_7^{eff}}(\frac{\mathrm{T_2}(\hat{\mathrm{s}}_0)}{\mathrm{A_1}(\hat{\mathrm{s}}_0)}(1-\hat{\mathrm{m}}_\mathrm{V}) + \frac{\mathrm{T_1}(\hat{\mathrm{s}}_0)}{\mathrm{V}(\hat{\mathrm{s}}_0)}(1+\hat{\mathrm{m}}_\mathrm{V}))$$

- Model-dependent studies $\Longrightarrow$ small FF-related uncertainties in $\hat{s}_0$ [Burdman '98]
- HQET provides a symmetry argument why the uncertainty in $\hat{s}_0$ is small. In leading order in $1/m_B$, $1/E$ ($E = \frac{m_B^2+m_{K^*}^2-q^2}{2m_B}$) and $O(\alpha_s)$:

$$\frac{T_2}{A_1} = \frac{1+\hat{m}_V}{1+\hat{m}_V^2-\hat{s}}(1-\frac{\hat{s}}{1-\hat{m}_V^2}); \quad \frac{T_1}{V} = \frac{1}{1+\hat{m}_V}$$

- No hadronic uncertainty in $\hat{s}_0$ [AA, Ball, Handoko, Hiller '99]: $C_9^{eff}(\hat{s}_0) = -\frac{2m_bM_B}{s_0}C_7^{eff}$

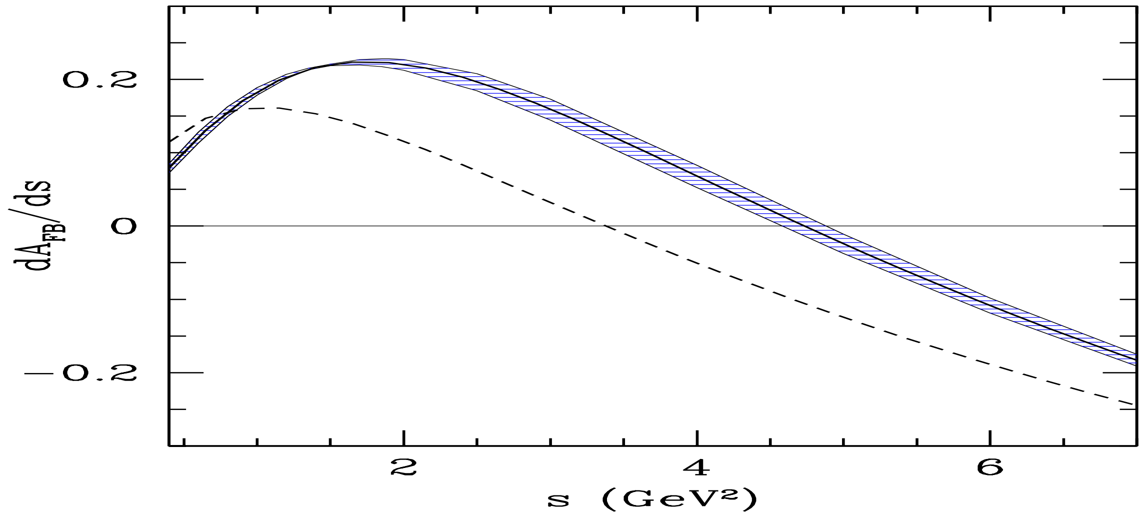
$O(\alpha_s)$ corrections to FB-Asymmetry in $B \to K^* \ell^+ \ell^-$

- $O(\alpha_s)$ corrections to the LEET-symmetry relations lead to substantial perturbative shift in $\hat{s}_0$ [Beneke, Feldmann, Seidel '01]

$$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff}(1 + \frac{\alpha_s C_F}{4\pi}[\ln \frac{m_b^2}{\mu^2} - L] + \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp}{\xi_\perp(s_0)})$$

[AA, A.S. Safir (hep-ph/02054)]

H



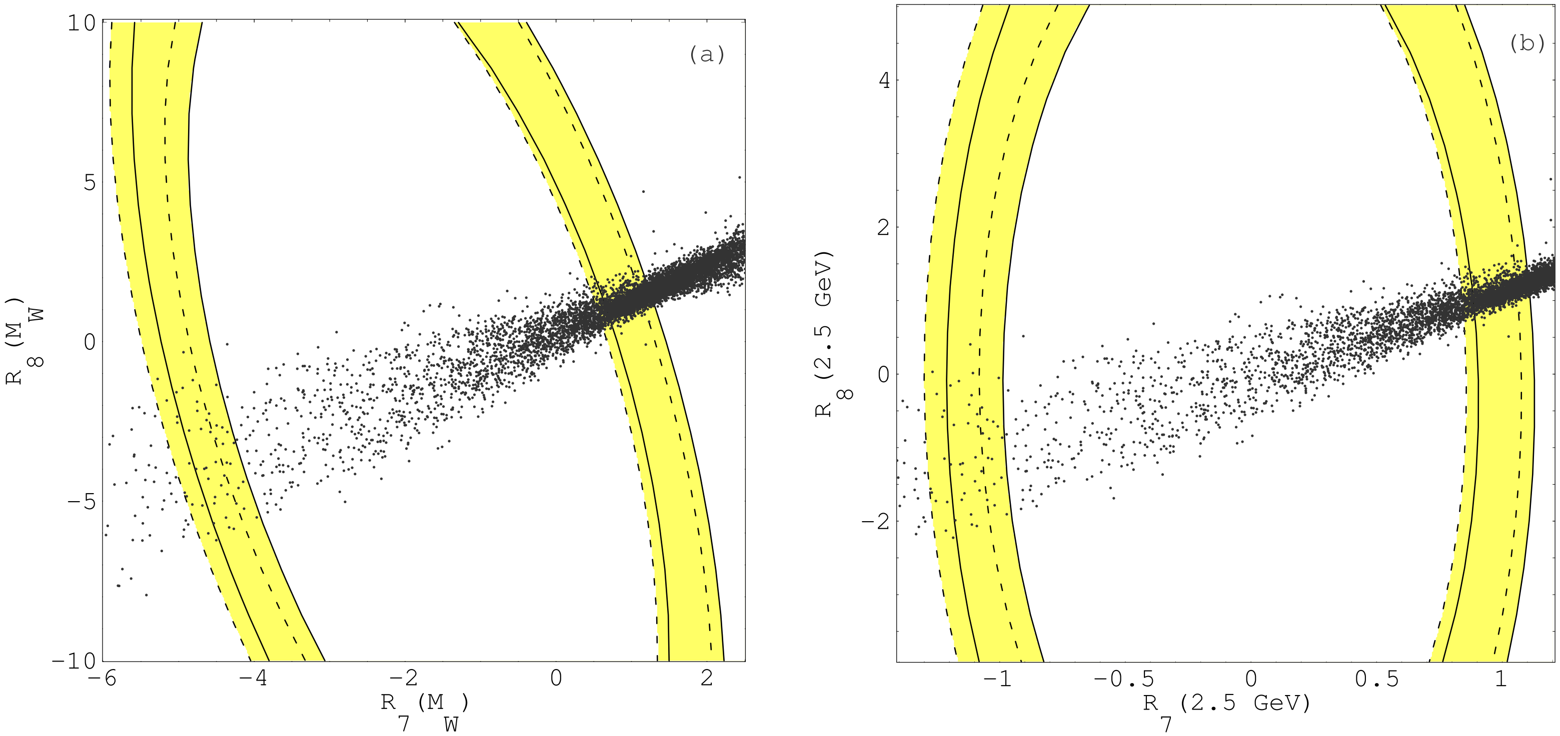
Forward-backward asymmetry $dA_{FB}(B \to K^* l^+ l^-)/ds$ at next-to-leading order (solid center line) and leading order (dashed)

A Model-independent Analysis of $B \to X_s \gamma$ & $B \to X_s \ell^+ \ell^-$

- Assume $\mathcal{H}_{eff}^{\rm SM}$ a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM [BSM] physics only in $C_7(\mu_W)$,$C_8(\mu_W)$, $C_9(\mu_W)$, and $C_{10}(\mu_W)$
- BSM Coefficients: $R_7 - 1$, $R_8 - 1$, $C_9^{\rm NP}$, & $C_{10}^{\rm NP}$
- Define: $R_{7,8}(\mu_W) \equiv \frac{C_{7,8}^{\rm tot}(\mu_W)}{C_{7,8}^{\rm SM}(\mu_W)}$
  with $C_{7,8}^{\rm tot}(\mu_W) = C_{7,8}^{\rm SM}(\mu_W) + C_{7,8}^{\rm NP}(\mu_W)$
- Set the scale $\mu_W = M_W$, and use RGE to evolve
  $$R_{7,8}(\mu_W) \to R_{7,8}(\mu_b) = \frac{A_{7,8}^{\rm tot}(\mu_b)}{A_{7,8}^{\rm SM}(\mu_b)}$$
- Impose constraints from $R_7(\mu_b)$ and $R_8(\mu_b)$ from $B \to X_s \gamma$ Data
- Use Data on $B \to (X_s, K^*, K)\ell^+\ell^-$ BRs to constrain $C_9^{\rm NP}$ and $C_{10}^{\rm NP}$
- Two-fold ambiguity due to the sign of C_7^{eff} can be resolved by data on $\boldsymbol{B \to (X_s, K, K^*)\ell^+\ell^-}$

Simulation of $B \to X_s\gamma$ in SUSY-MFV Models

- 90% C.L. bounds in the $[R_7(\mu), R_8(\mu)]$ plane from the $\mathcal{B}(B \to X_s\gamma)$
  $\mu = m_W$ (left-hand plot); $\mu = 2.5$ GeV (right-hand plot)



$$A_7^{\rm tot} - \text{negative} : -0.37 \leq A_7^{{\rm tot},<0}(2.5\ \text{GeV}) \leq -0.17$$

$$A_7^{\rm tot} - \text{positive} : 0.21 \leq A_7^{{\rm tot},>0}(2.5\ \text{GeV}) \leq 0.43$$

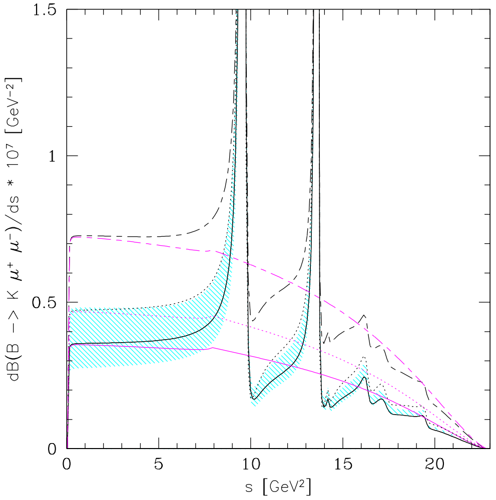
Dilepton mass-Spectrum in $\bar{B} \to (K, K^*)\ell^+\ell^-$ in SM and SUSY

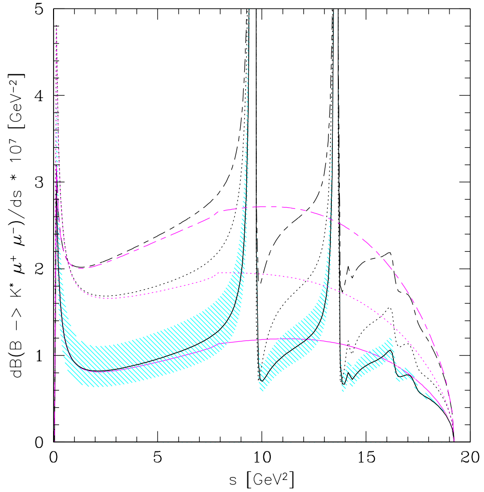
AA, Ball, Handoko, Hiller; hep-ph/9910221

- NP contributions coded in $R_i(\mu)$; $i = 7,\ 9,\ 10$

$$R_i(\mu) \equiv \frac{C_i^{\rm NP} + C_i^{\rm SM}}{C_i^{\rm SM}}$$

  - SM (solid); SUGRA $[R_7 = -1.2]$ (dots);
  - MIA $[R_7 = -0.83,\ R_9 = 0.92,\ R_{10} = 1.6]$ (dashed)

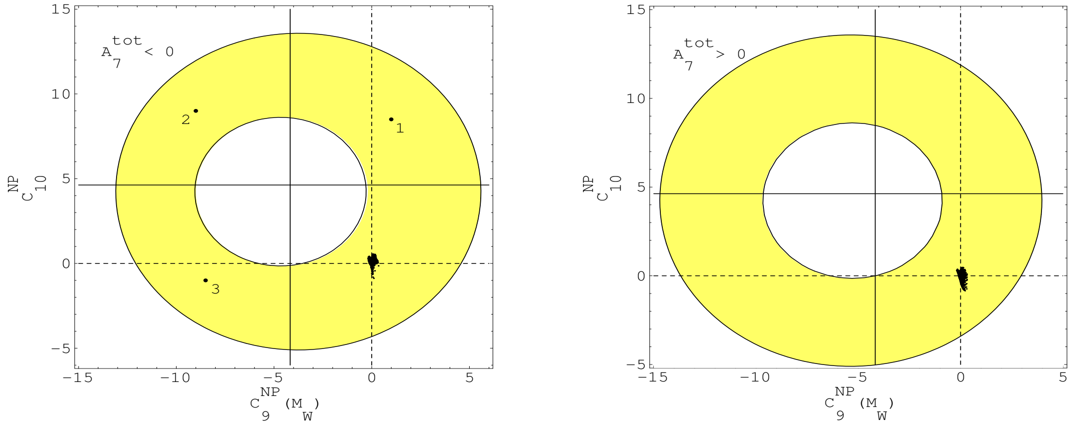




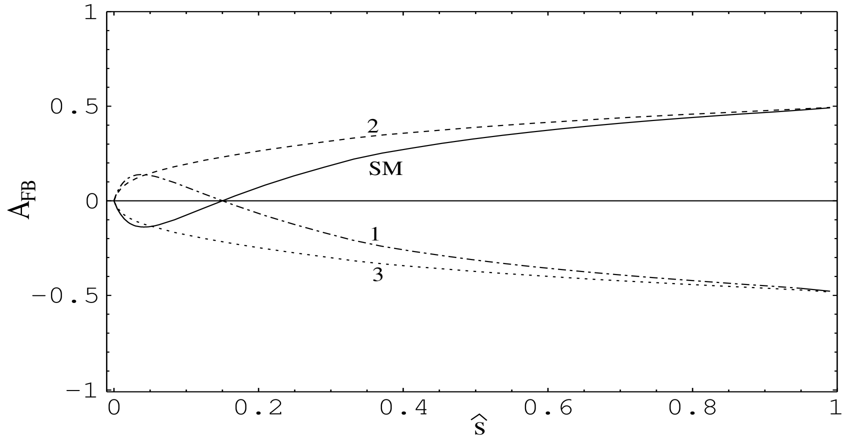
Combined analysis of $B \to X_s\gamma$ & $B \to (X_s, K, K^*)\ell^+\ell^-$

[A.A., Lunghi, Greub, Hiller; DESY 01-217; hep-ph/0112300]

- Constraints from radiative and semileptonic rare decays (Points: SUSY-MFV Model)



- FB asymmetry for $\bar{B} \to X_s\ell^+\ell^-$, corresponding to the points indicated above

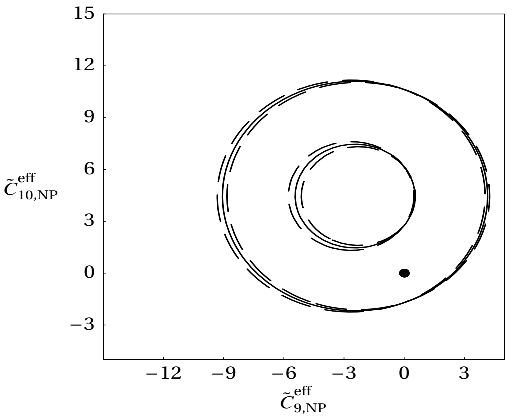


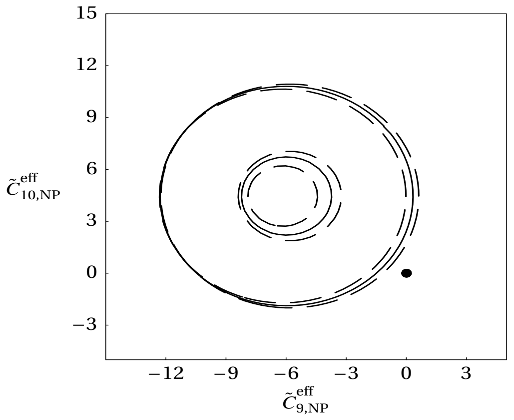
First hints on the sign of the $B \to X_s\gamma$ amplitude

[Gambino, Haisch, Misiak; hep-ph/0410155]

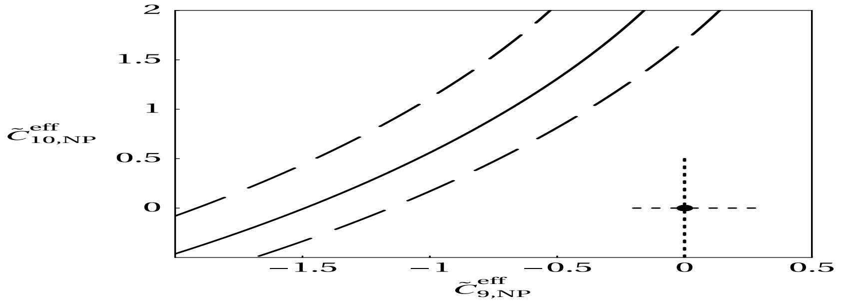
90% C.L. constraints from $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

C_7 SM-like (left frame) C_7 opposite sign (right frame)

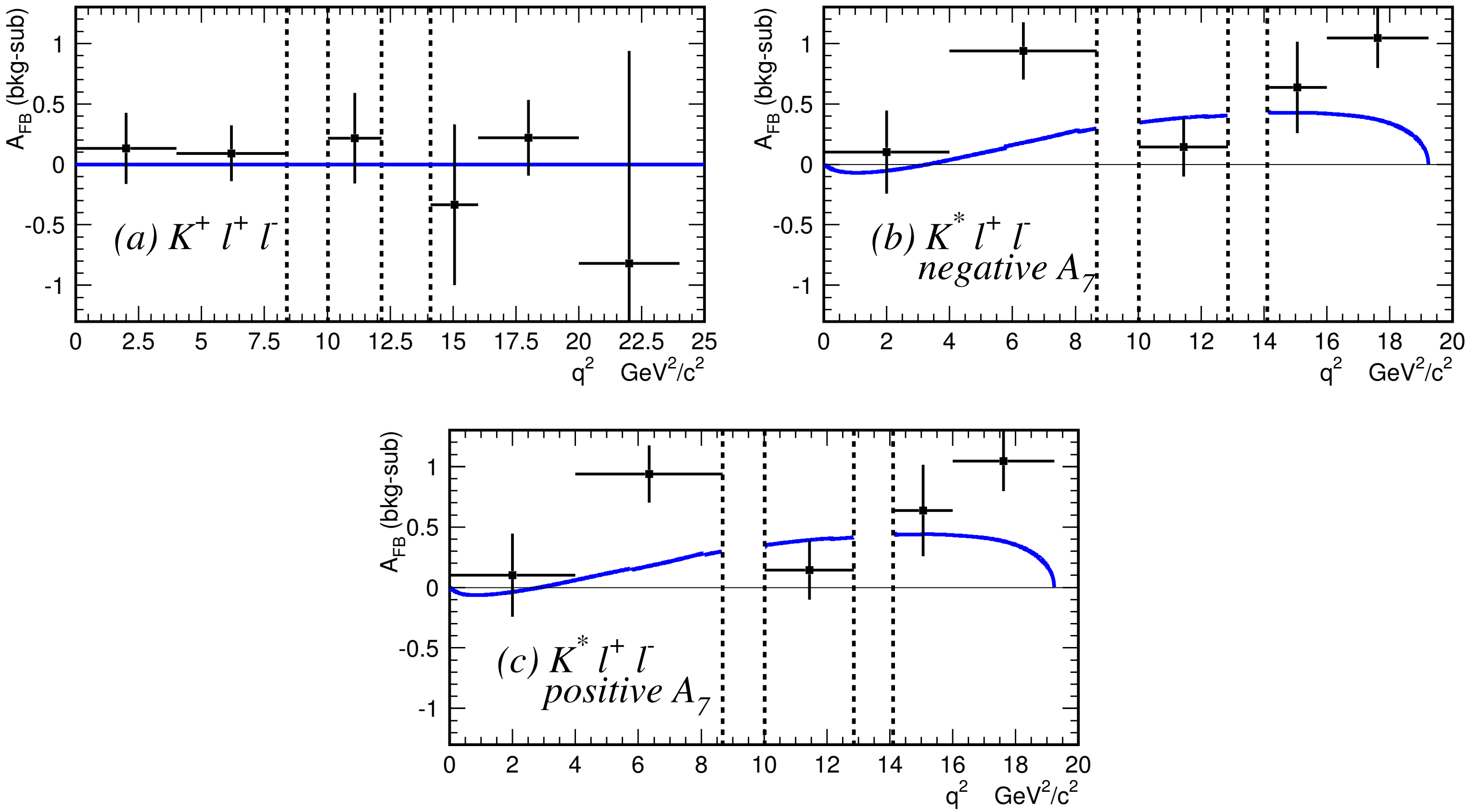




Surroundings of the origin in the right frame above; dashed lines: MFV-MSSM



Belle FB Asymmetry Distributions (EPS 2005)



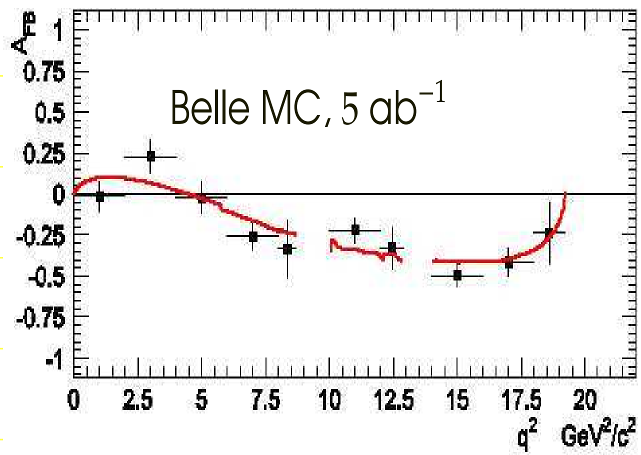
Best Fits

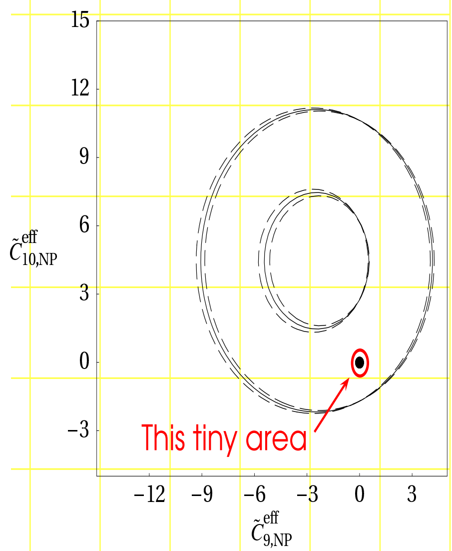
- $A_7 = -0.33$: $A_9/A_7 = -15.3^{+3.4}_{-4.8}$; $A_{10}/A_7 = 10.3^{+5.2}_{-3.5}$
- $A_7 = +0.33$: $A_9/A_7 = -16.3^{+3.7}_{-5.7}$; $A_{10}/A_7 = 11.1^{+6.0}_{-3.9}$
- SM: $A_7 = -0.33$; $A_9/A_7 = -12.3$; $A_{10}/A_7 = 12.8$

Prospects of precise determination of C_9, C_{10} at Super-B Factory

Extracting C_9 and C_{10} from $B \to K^* \ell^+ \ell^-$

- Precise determination of C_9 and C_{10} is possible
- $\Delta C_9/C_9 \sim 11\%$, $\Delta C_{10}/C_{10} \sim 13\%$ at 5 ab^{-1}, C_7 fixed from $b \to s\gamma$
  - Current branching fraction / background extrapolated
  - Fit to 2-dim q^2 vs angular distribution, not simple A_{FB}
  - Systematic error is neglected





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LHC-B MC Studies

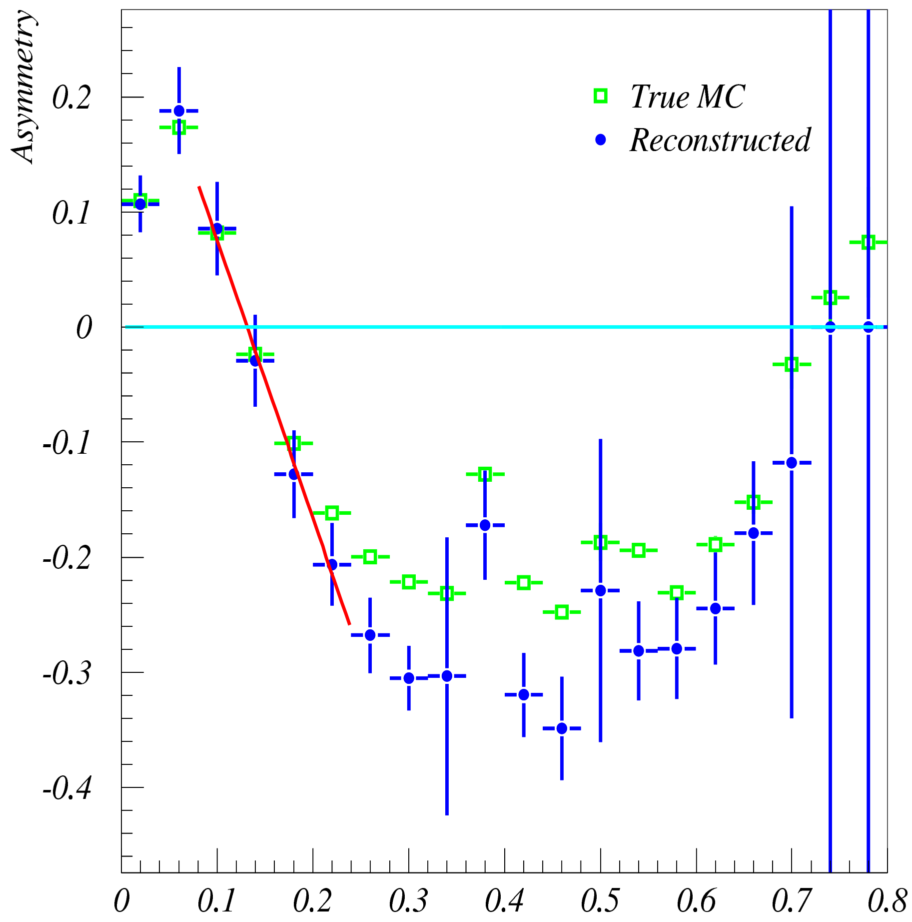


Figure 4: FB Asymmetry versus $\hat{s}$ **for** $B \stackrel{\hat{s}}{\longrightarrow} \mu^+\mu^-K^*$ **(from Koppenburg)**

$B_s \to \mu^+\mu^-$ in SM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}{}^* V_{tb} \sum_i \left[C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu) \right]$$

$$\begin{aligned} \mathcal{O}_{10} &= (\bar{s}_\alpha \gamma^\mu P_L b_\alpha)\,(\bar{l}\gamma_\mu\gamma_5 l)\,, & \mathcal{O}_{10}' &= (\bar{s}_\alpha \gamma^\mu P_R b_\alpha)\,(\bar{l}\gamma_\mu\gamma_5 l) \\ \mathcal{O}_S &= m_b\,(\bar{s}_\alpha P_R b_\alpha)\,(\bar{l}l)\,, & \mathcal{O}_S' &= m_s\,(\bar{s}_\alpha P_L b_\alpha)\,(\bar{l}l) \\ \mathcal{O}_P &= m_b\,(\bar{s}_\alpha P_R b_\alpha)\,(\bar{l}\gamma_5 l)\,, & \mathcal{O}_P' &= m_s\,(\bar{s}_\alpha P_L b_\alpha)\,(\bar{l}\gamma_5 l) \end{aligned}$$

$$\begin{aligned} \mathrm{BR}\left(\bar{B}_s \to \mu^+\mu^-\right) =& \frac{G_F^2\alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{ts}{}^* V_{tb}|^2 \sqrt{1-4\hat{m}_\mu^2} \\ & \times \left[\left(1-4\hat{m}_\mu^2\right) |F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2 \right] \end{aligned}$$

where $\hat{m}_\mu = m_\mu / m_{B_s}$ and

$$F_{S,P} = m_{B_s}\left[\frac{C_{S,P}m_b - C_{S,P}' m_s}{m_b + m_s}\right], \qquad F_{10} = C_{10} - C_{10}'$$

$$\mathrm{BR}\left(\bar{B}_s \to \mu^+\mu^-\right)_{\mathrm{SM}} = (3.46 \pm 1.5) \times 10^{-9} \quad [\mathrm{Buchalla, Buras}]$$

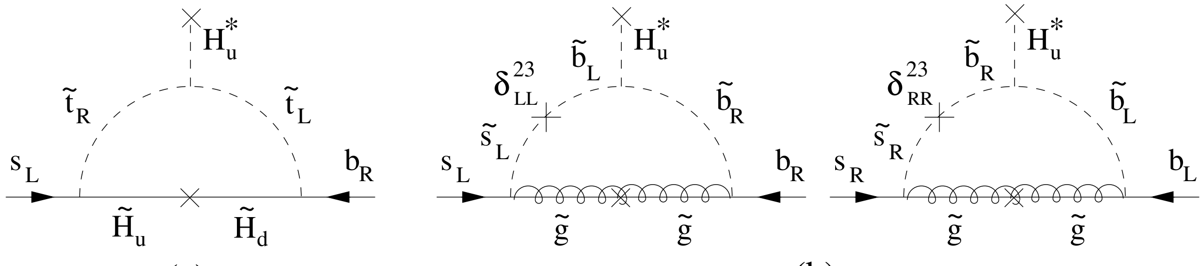
$$f_{B_s} = (230 \pm 30)\ \mathrm{MeV}$$

$B_s \to \mu^+\mu^-$ in Supersymmetric Models

- The decay $B_s \to \mu^+\mu^-$ probes essentially the Higgs sector of Supersymmetry, a type-II two-Higgs doublet model; One Higgs field (H_u) couples to the up-type quarks, the other (H_d) couples to the down-type quarks

$$\mathcal{L} = \overline{Q} Y_U U_R H_u + \overline{Q}_L Y_D D_R H_d$$

- Supersymmetry does not have discrete symmetries to protect the alignment of the Higgs boson interaction eigenbasis with the fermion mass eigenbasis; Higgs-induced FCNC interactions are generated through loops



- As H_u gets a VEV (v_u), it contributes an off-diagonal piece to the down-type fermion mass matrix, mixing s_L and b_L by an angle θ

$$\sin\theta = y_b \epsilon v_u / m_b; \quad \text{as } m_b = y_b v_d, \ \sin\theta = \epsilon \tan\beta$$

- $\mathcal{A}(b\bar{s} \to \mu^+\mu^-) \simeq \sin\theta \mathcal{A}(b\bar{b} \to \mu^+\mu^-) \propto \tan\beta / \cos^2\beta \Longrightarrow \tan^3\beta$ for large-$\tan\beta$

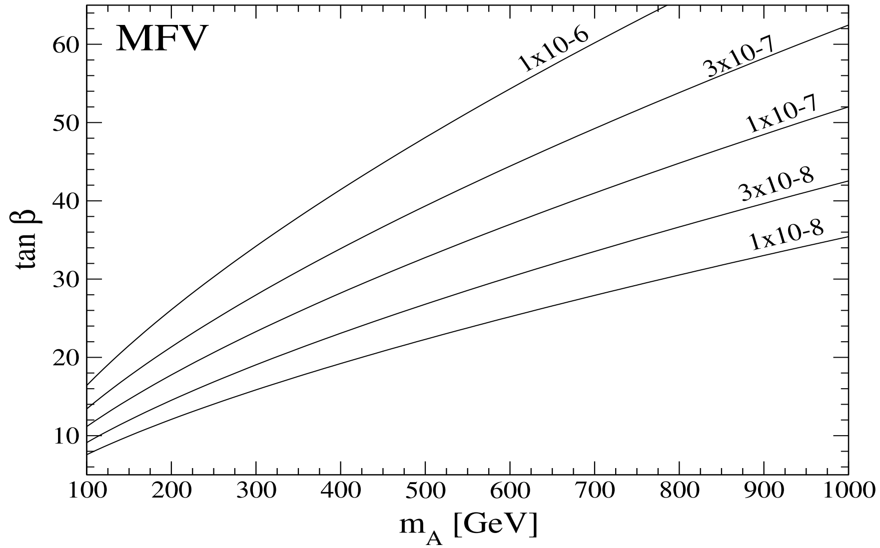
$B_s \to \mu^+\mu^-$ in Minimal Flavor Violation SUSY Models

- Higgsino contribution to $\mathcal{B}(B_s \to \mu^+\mu^-)$ [Babu, Kolda;...]

$$\mathcal{B}(B_s \to \mu\mu) \simeq \frac{G_F^2}{8\pi}\, \eta_{\rm QCD}^2 m_{B_s}^3 f_{B_s}^2 \tau_{B_s} m_b^2\, m_\mu^2 \left(\frac{\tan^2\beta}{\cos^4\beta}\right)\left(\frac{\kappa_{\widetilde{H}}^2}{m_A^4}\right).$$

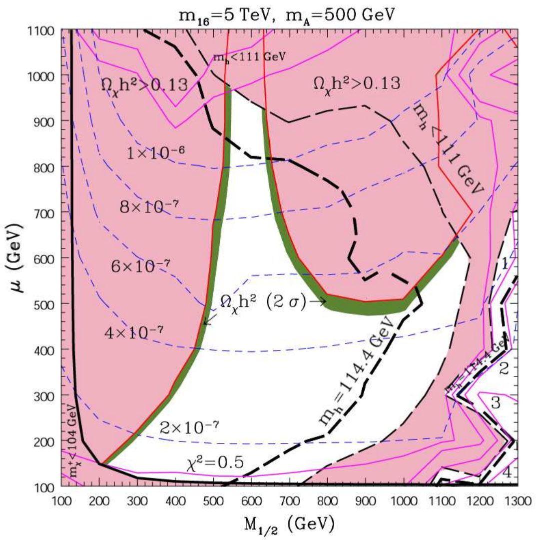
- $\eta_{\rm QCD} \simeq 1.5$ is the QCD correction due to the RG between the SUSY and B_s scales

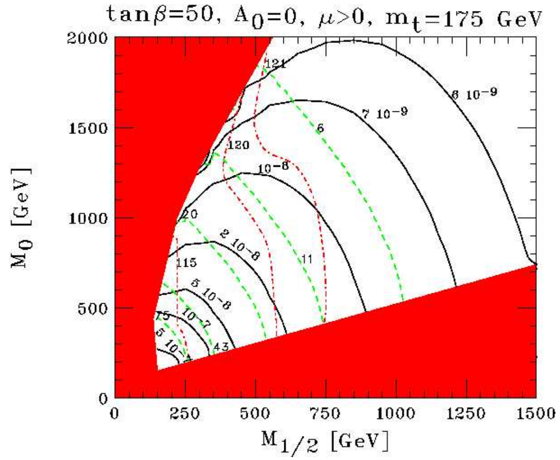
$$\kappa_{\widetilde{H}} = -\frac{G_F\, m_t^2\, V_{ts}V_{tb}}{4\sqrt{2}\pi^2 \sin^2\beta}\, \mu A_t\, f(\mu^2, m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2)$$



Constraints from $\mathcal{B}(B_s \to \mu^+\mu^-)$ on SUSY Models

- **CDF $B_s^0/B_d^0 \to \mu^+\mu^-$ Limits [hep-ex/0508036]:**
  $\mathcal{B}(B_s \to \mu^+\mu^-) < 1.5 \times 10^{-7} (2.0 \times 10^{-7})$ at 90%(95%) CL
  $\mathcal{B}(B_d \to \mu^+\mu^-) < 3.9 \times 10^{-8} (5.1 \times 10^{-8})$ at 90%(95%) CL
- **D0 $B_s^0 \to \mu^+\mu^-$ Limits [D0note 4733-Conf (2005)]:**
  $\mathcal{B}(B_s \to \mu^+\mu^-) < 3.2 \times 10^{-7} (4.0 \times 10^{-7})$ at 90%(95%) CL
  $\Longrightarrow$ **complementary limits on models of BSM physics, such as MSUGRA [Dedes et al., hep-ph/0108037], SO(10) [Dermisek et al., hep-ph/0304101; Foster et al., hep-ph/0506146] and MFV models [Bobeth et al., hep-ph/0505110]**





Summary

- All current measurements involving FCNC processes (decay rates and distributions) are in agreement with the SM expectations
- Rare B-decays and B^0 - $\overline{B^0}$ mixings have made a great impact on the determination of the CKM matrix elements in the third row of $V_{\rm CKM}$; In particular
  $B \to X_s\gamma \implies V_{ts} = -(46.0 \pm 8.0) \times 10^{-3}$
  $B \to (\rho, \omega, K^*)\gamma \implies |\frac{V_{td}}{V_{ts}}| = 0.200^{+0.026}_{-0.025}{}^{+0.038}_{-0.029}$
- A number of benchmark measurements remain to be done. These include, among others, $\mathcal{B}(B_s \to \mu^+\mu^-)$ and ΔM_{B_s}, which will be carried out at Fermilab and LHC; Correlations between these and other Rare B/K-decays crucial to disentangle BSM physics in the flavour sector
- Discovery of SUSY at LHC but continued absence of observable effects in FCNC and CPV beyond SM would point to a flavour-blind SUSY (such as mSUGRA,MFV)
- However, data on CPV in $\boldsymbol{b \longrightarrow s\bar{s}s}$ penguins puzzling; need to clarify this effect experimentally - a motivation to build a Super-B factory
- Hope that the synergy of high energy frontier and low energy precision physics, which worked so well in piecing together the SM, will continue to hold sway in the LHC-era, providing valuable information about the flavour aspects of the BSM physics