NEUTRINO PHYSICS

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Lecture AB-I. Properties of neutrinos.

Lecture AB-II. Rare meson decays mediated by Majorana neutrinos.

Lecture AB-III. Heavy Majorana neutrinos in high-energy lepton-proton and proton-proton collisions.

Lecture AB-I
PROPERTIES OF NEUTRINOS

OUTLINE

- **Brief history**
- **Neutrino flavors**
- **Chirality and helicity**
- **Observation of neutrino oscillations: nonzero neutrino masses and mixing**
- **Limits on the neutrino masses**
- **The nature of neutrino masses: Dirac or Majorana?**
- **The “seesaw” mechanism for neutrino masses**
- **Conclusion**

Brief history

The first evidence for the neutrinos appeared in the study of nuclear beta decays (1920's):

$$(A,Z) \rightarrow (A,Z \pm 1) + e^{\mp} + ?(\textit{nothing else visible})$$

with a *continuum* $e^{\mp}$ spectrum ranging from m_e up to

$$E_{e\max} \simeq Q \equiv M_i - M_f .$$

Niels Bohr speculated about the possibility of energy *non*conservation.

In December 1930 Wolfgang Pauli suggested the existence of a unobserved *neutral* and *very light* particle with spin ½ taking into account *conservation* of electric charge, energy ($Q - E_{e\max} \simeq 0$) and angular momentum (and statistics). This new particle was named *neutrino* ("small neutron" in Italian) by Enrico Fermi in 1932.

In 1934 E. Fermi proposed the first theory of beta decay and weak interactions in general. The interaction Lagrangian of Fermi's theory

$$L_{weak} = -\frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu$$

is the product of two weak currents at the same space-time point, i. e., 4-fermion interaction with *zero* radius (in fact, very short ranged).

The coupling strength (Fermi constant)

$$G_F = 1.16637(1)\times 10^{-5}\ \text{GeV}^{-2} \simeq \frac{10^{-5}}{m_p^2}.$$

Fermi's theory predicted the existence of *anti*neutrinos and successfully described beta decays:

$$(A,Z) \to (A,Z-1) + e^+ + \nu_e,$$

$$(A,Z) \to (A,Z+1) + e^- + \overline{\nu}_e,$$

$$n \to p + e^- + \overline{\nu}_e\ (\tau_n = 886.7\ \text{s} \simeq 15\ \text{min}).$$

It also predicted other *weak* processes, in particular the *inverse* beta decays such as:

$$\overline{\nu}_e + p \to e^+ + n,$$

$$\nu_e + n \to e^- + p.$$

The *direct* detection of (anti)neutrinos was performed by C. Cowan and F. Reines in 1956 through the coincident observation of a positron and a photon (in fact, with a few μsec delay) emitted in subsequent reactions

$$\overline{\nu}_e + p \to e^+ + n \Rightarrow n + {}^{112}\text{Cd} \to {}^{113}\text{Cd} + \gamma.$$

They used the nuclear reactor (Savannah River, South Carolina) as an antineutrino *source* and 200 liters of water with 40 kg of dissolved $CdCl_2$ as a *detector*.

Frederick Reines was awarded the 1995 Nobel Prize in Physics "for the detection of the neutrino".

The detailed studies of beta decays established the $V-A$ structure of weak interactions leading to parity violation (C. N. Yang and T.-D. Lee received the 1957 Nobel Prize in Physics "for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles").

The modern theory of weak interactions is a part of the non-abelian gauge theory of electroweak interactions developed in the 1960's by Sheldon L. Glashow, Abdus Salam, Steven Weinberg (the 1979 Nobel Prize in Physics "for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current").

Renormalizability of the standard electroweak model (SM) based on the gauge group $SU(2)_L \times U(1)_Y$ (spontaneously broken to $U(1)_{em}$) was proved by

Gerardus ‘t Hooft and Martinus J. G. Veltman in the beginning of 1970’s (the 1999 Nobel Prize in Physics “for elucidating the quantum structure of electroweak interactions in physics”).

Neutrino flavors

Up to now three distinct types of neutrinos (and corresponding antineutrinos) have been detected: ν_e, ν_μ, ν_τ $(\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)$.

Electron neutrinos **and antineutrinos are produced in beta becays (see below) and in other decays, e. g.:**

$$\pi^+ \to e^+ + \nu_e,\ \pi^- \to e^- + \bar{\nu}_e,$$

$$K^+ \to \pi^0 + e^+ + \nu_e,\ K^- \to \pi^0 + e^- + \bar{\nu}_e,$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu,\ \mu^- \to e^- + \bar{\nu}_e + \nu_\mu.$$

It was supposed that $\nu \neq \bar{\nu}$**:**

$\bar{\nu}_e + {}^{37}Cl \not\to {}^{37}Ar + e^-$ **(R. Davis, Jr., 1952),**

$\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$ **(R. Davis, Jr., D. S. Harmer, and K. C. Hoffmann, 1968) [the experiment was proposed by B. Pontecorvo in 1946].**

***Muon (anti)neutrinos*:**

$$\pi^+ \to \mu^+ + \nu_\mu,\ \pi^- \to \mu^- + \bar{\nu}_\mu,$$

$$K^+ \to \mu^+ + \nu_\mu,\ K^- \to \mu^- + \bar{\nu}_\mu,$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu,\ \mu^- \to e^- + \bar{\nu}_e + \nu_\mu.$$

The experimental evidence that $\nu_\mu \neq \nu_e$:

$$\nu_\mu + N \to \mu^- + X \text{ but } \nu_\mu + N \not\to e^- + X$$

(L. Lederman, M. Schwarz, and J. Steinberger, 1962).

***Tau neutrinos*:**

$$\tau^- \to e^- + \bar{\nu}_e + \nu_\tau, \quad \tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau,$$

$$\tau^- \to \nu_\tau + \text{hadrons}.$$

The third charged lepton τ^- was discovered in 1975 at Stanford. *Direct evidence for the tau neutrino* was obtained in 2001 by the DONUT collaboration at Fermilab through the observation of τ-appearance in nuclear emulsions:

$$\nu_\tau + N \to \tau + X.$$

The type of a neutrino ν_ℓ is called *flavor*. In the SM there are three neutrino flavors ($\ell = e, \mu, \tau$) associated with the corresponding charged leptons in three electroweak-isospin doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L.$$

Neutrinos ν_ℓ have two types of interactions mediated by charged $W^\pm$ bosons (charged current interactions) and by a neutral Z^0 boson (neutral current interactions):

$$L_{CC} = -\frac{g}{\sqrt{2}} \sum_\ell \left[W_\mu^- \left(\bar{\ell} \gamma^\mu \frac{1-\gamma^5}{2} \nu_\ell \right) + W_\mu^+ \left(\bar{\nu}_\ell \gamma^\mu \frac{1-\gamma^5}{2} \ell \right) \right],$$

$$L_{NC} = -\frac{g}{\cos\theta_W} Z_\mu \sum_\ell \left(\bar{\nu}_\ell \gamma^\mu \frac{1-\gamma^5}{2} \nu_\ell \right),$$

where the coupling constant $g = e/\sin\theta_W$, θ_W is the weak (Weinberg) angle ($\sin^2\theta_W \simeq 0.23$), e is the positron electric charge.

By definition, the flavor of a neutrino is the type of the charged lepton that is connected to the same charged current vertex: $W^+ \to \ell^+ + \nu_\ell, \quad W^- \to \ell^- + \bar{\nu}_\ell$.

Number of light neutrinos

The partial width of the decay $Z \to \nu_\ell + \bar{\nu}_\ell$ is calculable in the SM:

$$\Gamma_\nu = \frac{G_F}{\sqrt{2}} \frac{m_Z^3}{12\pi} = 165.9\ \mathrm{MeV}.$$

The invisible width of the decay into all neutrino final states:

$$\Gamma_{\rm inv} = N_\nu \Gamma_\nu = 499.0 \pm 1.5\ \mathrm{MeV}.$$

It is obtained experimentally from studies of single-photon events from the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ or by subtracting the contribution of all visible channels ($Z \to \ell^+\ell^-$, $Z \to q\bar{q}$; $\ell = e, \mu, \tau$; $q = u, d, s, c, b$) from the measured total width, $\Gamma_{\rm inv} = \Gamma_Z - \Gamma_{\rm vis}$.

The number of the *light active* neutrinos (that have the usual electroweak interactions) is (see Fig. 1):

$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} = 3.01.$$

PDG (2004):

$N_\nu = 2.994 \pm 0.012$ (**SM fits to LEP data**),

$N_\nu = 2.92 \pm 0.07$ (**Direct measurement of $\Gamma_{\rm inv}$**).

Annihilation Cross Section Near M_Z

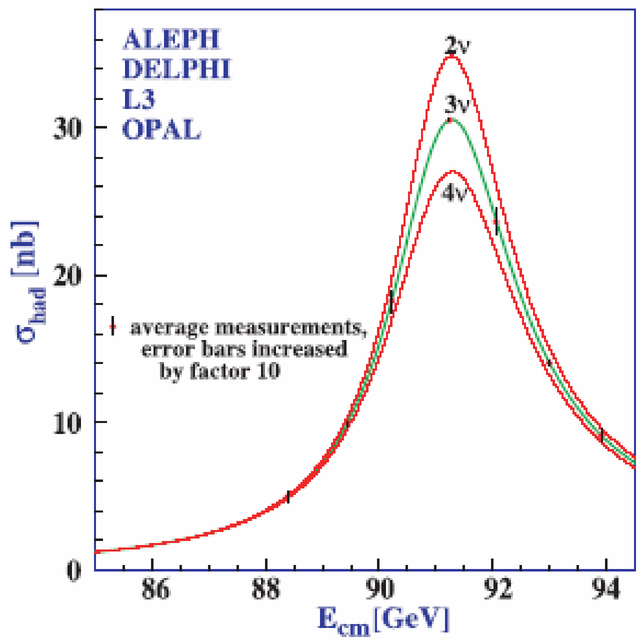

Figure 40.8: Combined data from the ALEPH, DELPHI, L3, and OPAL Collaborations for the cross section in e^+e^- annihilation into hadronic final states as a function of the center-of-mass energy near the Z pole. The curves show the predictions of the Standard Model with two, three, and four species of light neutrinos. The asymmetry of the curve is produced by initial-state radiation. Note that the error bars have been increased by a factor ten for display purposes. References:

ALEPH: R. Barate *et al.*, Eur. Phys. J. **C14**, 1 (2000).
DELPHI: P. Abreu *et al.*, Eur. Phys. J. **C16**, 371 (2000).
L3: M. Acciarri *et al.*, Eur. Phys. J. **C16**, 1 (2000).
OPAL: G. Abbiendi *et al.*, Eur. Phys. J. **C19**, 587 (2001).
Combination: The Four LEP Collaborations (ALEPH, DELPHI, L3, OPAL) and the Lineshape Sub-group of the LEP Electroweak Working Group, hep-ph/0101027.

(Courtesy of M. Grünewald and the LEP Electroweak Working Group, 2003)

Fig. 1.

Chirality and helicity

In 1950's it was discovered that all neutrinos have (*within experimental uncertainties*) spin antiparallel to their momentum, while for all antineutrinos spin and momentum are parallel. This is a consequence of the $V - A$ structure of the weak currents:

$$L_{CC} = -\frac{g}{\sqrt{2}}\left(W_\mu^- j^{-\mu} + W_\mu^+ j^{+\mu}\right),$$

$$j^{-\mu} = \sum_\ell \overline{\ell}_L \gamma^\mu \nu_{\ell L}, \quad j^{+\mu} = \sum_\ell \overline{\nu}_{\ell L} \gamma^\mu \ell_L,$$

where

$$\psi_L = \frac{1-\gamma^5}{2}\psi, \quad \overline{\psi}_L = \overline{\psi_L} = \overline{\frac{1-\gamma^5}{2}\psi} = \overline{\psi}\frac{1+\gamma^5}{2}, \quad \overline{\psi} = \psi^+\gamma^0;$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \gamma^5\gamma^\mu = -\gamma^5\gamma^\mu, \quad \left(\gamma^5\right)^+ = \gamma^5, \quad \left(\gamma^5\right)^2 = I.$$

***Chirality* is eigenvalue of the operator γ^5:**

$$\gamma^5\psi_L = -\psi_L, \quad \gamma^5\psi_R = +\psi_R;$$

$$\psi_R = \frac{1+\gamma^5}{2}\psi, \quad \psi = \psi_L + \psi_R;$$

$$\bar{\psi}\gamma^\mu\partial_\mu\psi = \bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + \bar{\psi}_R\gamma^\mu\partial_\mu\psi_R,$$

$$\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R.$$

ψ_L and ψ_R are called *left-handed* and *right-handed* fields respectively.

The SM is a *chiral* gauge theory, since there are *L*-doublets and *R*-singlets,

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, e_R; \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R \text{ etc.},$$

which have *different* weak interactions.

The Dirac equation for a massive particle with 4-momentum $p^\mu = (E, \mathrm{p})$,

$$\left(\gamma^\mu p_\mu - m\right)\psi = 0,$$

can be decomposed as follows:

$$\hat{h}\psi_L = -\frac{E}{p}\psi_L + \frac{m}{p}\gamma^0\psi_R,$$

$$\hat{h}\psi_R = +\frac{E}{p}\psi_R - \frac{m}{p}\gamma^0\psi_L,$$

$$E = \left(m^2 + p^2\right)^{1/2}, \; p = |\mathbf{p}|,$$

where the *helicity* operator is

$$\hat{h} = \frac{\Sigma \cdot \mathbf{p}}{|\mathbf{p}|}, \quad \Sigma_k = \gamma^5\gamma^0\gamma^k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}.$$

***Helicity* is a conserved quantum number, in contrast with chirality that is conserved only in the massless limit. For *massless* particles helicity and chirality are identical:**

$$\hat{h}\psi_L = -\psi_L, \quad \hat{h}\psi_R = \psi_R.$$

For a massive particle, chirality states are *mixtures* of helicity states, and in the ultrarelativistic limit ($m/E \ll 1$)

$$\psi_L \simeq \psi_- + \frac{m}{2E}\psi_+, \quad \psi_R \simeq \psi_+ + \frac{m}{2E}\psi_-;$$

$$\hat{h}\psi_\pm = \pm\psi_\pm .$$

$$\psi_p^{(+)} = (2EV)^{-1/2} u(p,h) e^{-ip\cdot x}$$

$$u(p,h) = \begin{pmatrix} \sqrt{E+m}\, w \\ \sqrt{E-m}\, hw \end{pmatrix}, \quad \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{|\mathbf{p}|} w = hw,\ h = \pm 1.$$

$$P_L = \frac{1-\gamma^5}{2} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad P_R = \frac{1+\gamma^5}{2} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

$$u_L = P_L u = \frac{1}{2}\left(\sqrt{E+m} - h\sqrt{E-m}\right)\begin{pmatrix} w \\ -w \end{pmatrix},$$

$$u_R = P_R u = \frac{1}{2}\left(\sqrt{E+m} + h\sqrt{E-m}\right)\begin{pmatrix} w \\ w \end{pmatrix}.$$

For $\delta = \dfrac{m}{2E} \ll 1$:

$$u_L \simeq \sqrt{E}\left(\frac{1-h}{2} + \delta\frac{1+h}{2}\right)\begin{pmatrix} w \\ -w \end{pmatrix},$$

$$u_R \simeq \sqrt{E}\left(\frac{1+h}{2} + \delta\frac{1-h}{2}\right)\begin{pmatrix} w \\ w \end{pmatrix}.$$

The difference between chirality and helicity has important consequences. The impressive manifestation of the $V-A$ structure of the charged weak current is the helicity suppression in π decays:

$$R=\frac{\Gamma(\pi^+\to e^+ +\nu_e)}{\Gamma(\pi^+\to \mu^+ +\nu_\mu)}=\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)^2=1.28\times10^{-4},$$

$$R_{\exp}=\left(1.230\pm0.004\right)\times10^{-4}.$$

The decay amplitude is proportional to the admixture of negative helicity in the right-handed state of a charged lepton (see Fig. 2):

$$A(\pi^+\to \ell^+ +\nu_\ell)\sim m_\ell .$$

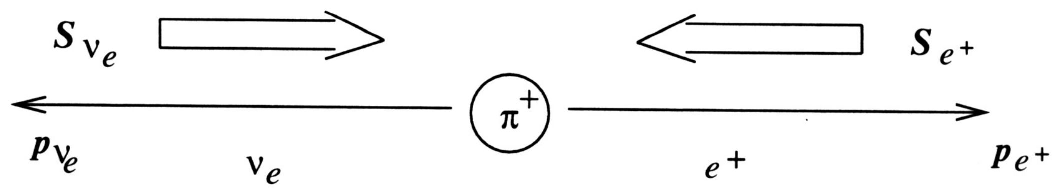

Fig. 2. The decay of a charge pion in its rest frame.

The chiral structure of weak interactions opens a possibility that the states ν and $\bar{\nu}$ are the two different helicity states of a Majorana neutrino ν_M that is identical to its antiparticle:

$$\nu = \nu_M (h = -1), \quad \bar{\nu} = \nu_M (h = +1).$$

In experiments with ultrarelativistic (anti)neutrinos, it is practically impossible to distinguish between Dirac ($\nu_D \neq \bar{\nu}_D$) and Majorana ($\nu_M \equiv \bar{\nu}_M$) neutrinos due to strong ($\sim (m/E)^2$) suppression of "wrong helicity" states.

Observation of neutrino oscillations: nonzero neutrino masses and mixing

The recent (1998-2002) observation of neutrino oscillations (predicted by B. Pontecorvo in 1957) implies that neutrinos are massive and mixed particles. The neutrino ν_ℓ of flavor $\ell = e, \mu, \tau$ is the superposition of neutrinos ν_i with definite masses m_i,

$$\nu_\ell = \sum_i U_{\ell i} \nu_i ,$$

where $U_{\ell i}$'s form the neutrino mixing matrix. The flavor neutrino ν_ℓ is created in association with the charged lepton ℓ^+ in the decay

$$W^+ \to \ell^+ + \nu_\ell \ .$$

The oscillation probability (see Fig. 3)

$$P(\nu_\alpha \to \nu_\beta) = \left|\langle \nu_\beta | \nu_\alpha(L)\rangle\right|^2 = \left|\sum_j U_{\beta j} U^*_{\alpha j} \exp\left(-i m_j^2 \frac{L}{2E}\right)\right|^2$$

$$= \sum_j \left|U_{\alpha j}\right|^2 \left|U_{\beta j}\right|^2 + 2\,\mathrm{Re} \sum_{j>k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \exp\left(-i \Delta m^2_{jk} \frac{L}{2E}\right),$$

where $\Delta m^2_{jk} = m_j^2 - m_k^2, E$ is the neutrino energy, L is the distance between a source and a detector. The expansion of the neutrino momentum $p_j = (E^2 - m_j^2)^{1/2}$ in $(m_j / E)^2$ has been used:

$$\exp(i p_j L) \simeq e^{iEL} \exp\left(-i \frac{m_j^2}{2E} L\right).$$

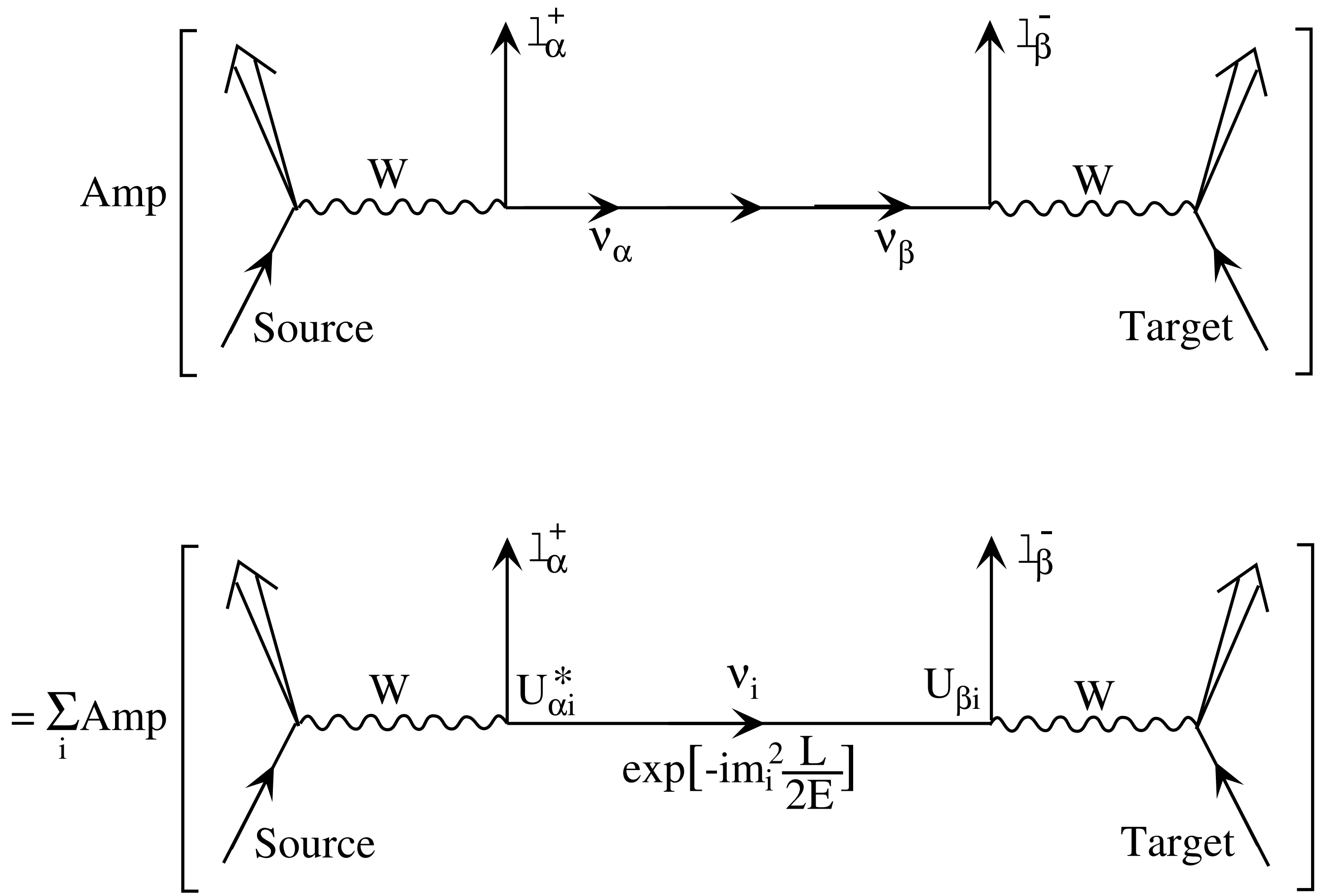

Fig. 3. Neutrino flavor change in vacuum.

In the simplest case of two-neutrino mixing

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix},$$

the oscillation probabilities (see Fig. 4)

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21}}{4E} L\right) = \frac{1}{2}\sin^2 2\theta\left[1 - \cos\left(2\pi \frac{L}{L_{\mathrm{osc}}}\right)\right];$$

$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta),$$

where the *oscillation length*

$$L_{\mathrm{osc}} = \frac{4\pi E}{\left|\Delta m^2_{21}\right|} = 2.48 \frac{E(\mathrm{MeV})}{\left|\Delta m^2_{21}\right|(\mathrm{eV}^2)}\ \mathrm{m}.$$

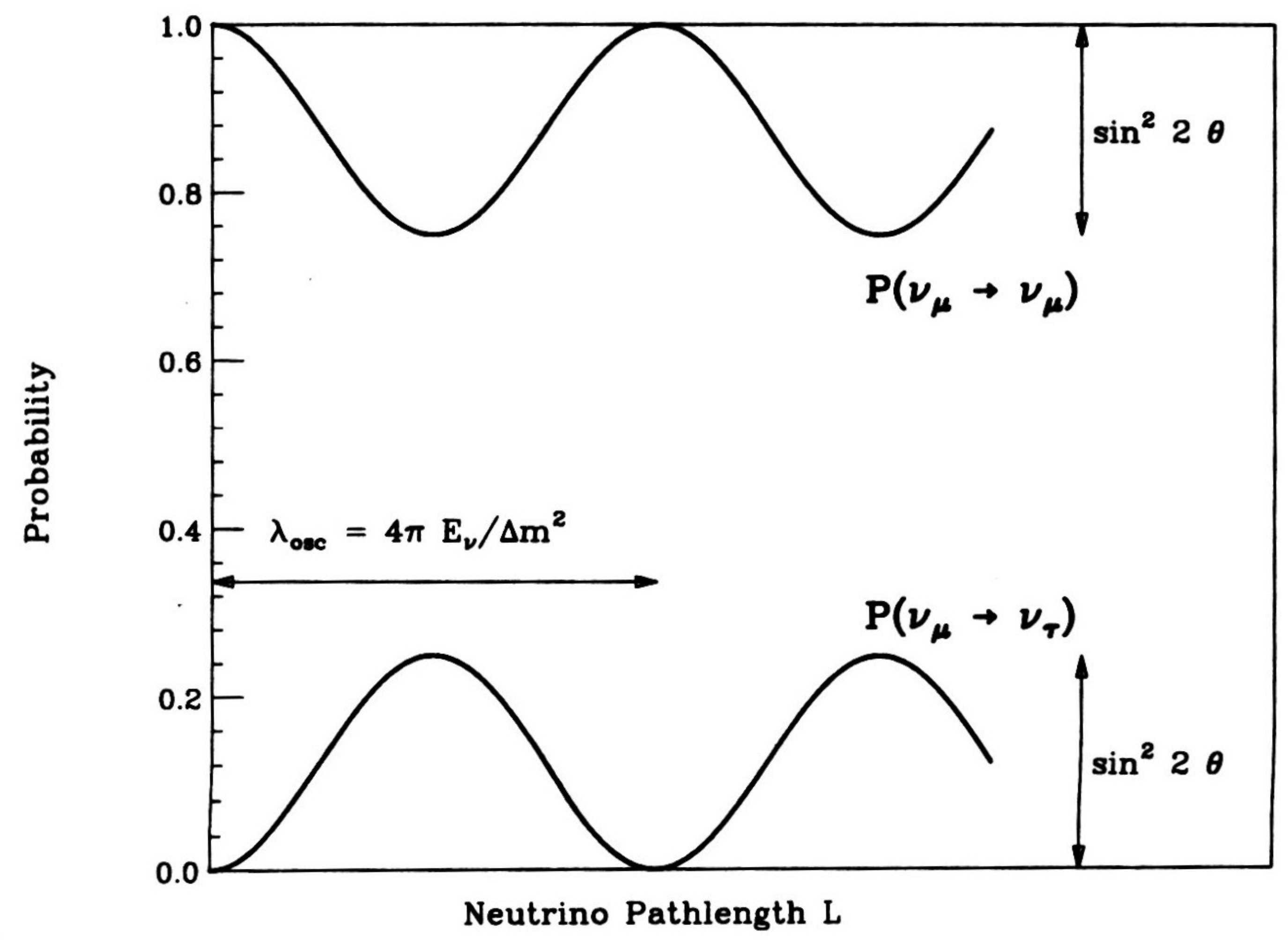
1.0
0.8
0.6
0.4
0.2
0.0
Probability
sin² 2 θ
P(ν_μ → ν_μ)
λ_osc = 4π E_ν/Δm²
P(ν_μ → ν_τ)
sin² 2 θ
Neutrino Pathlength L

Fig. 4.

Up to now the oscillations have been observed for
***solar* (Homestake, SAGE, GALLEX-GNO, SNO: $\nu_e \to \nu_\mu(\nu_\tau)$),**
***atmospheric* (Super-Kamiokande: $\nu_\mu \to \nu_\tau$) ,**
***reactor* (KamLAND: $\bar{\nu}_e \to \bar{\nu}_\mu$) [$\Leftrightarrow$ sol],**
and *accelerator* (K2K: $\nu_\mu \to \nu_\tau$)[$\Leftrightarrow$ atm]
neutrinos [for a review, see PDG-2004].

The *sinusoidal L/E dependence* of the survival probability

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2\theta \sin^2\left(1.27\Delta m^2(\mathrm{eV}^2)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right)$$

has been recently confirmed by KamLAND (hep-ex/0406035, see Fig. 5).

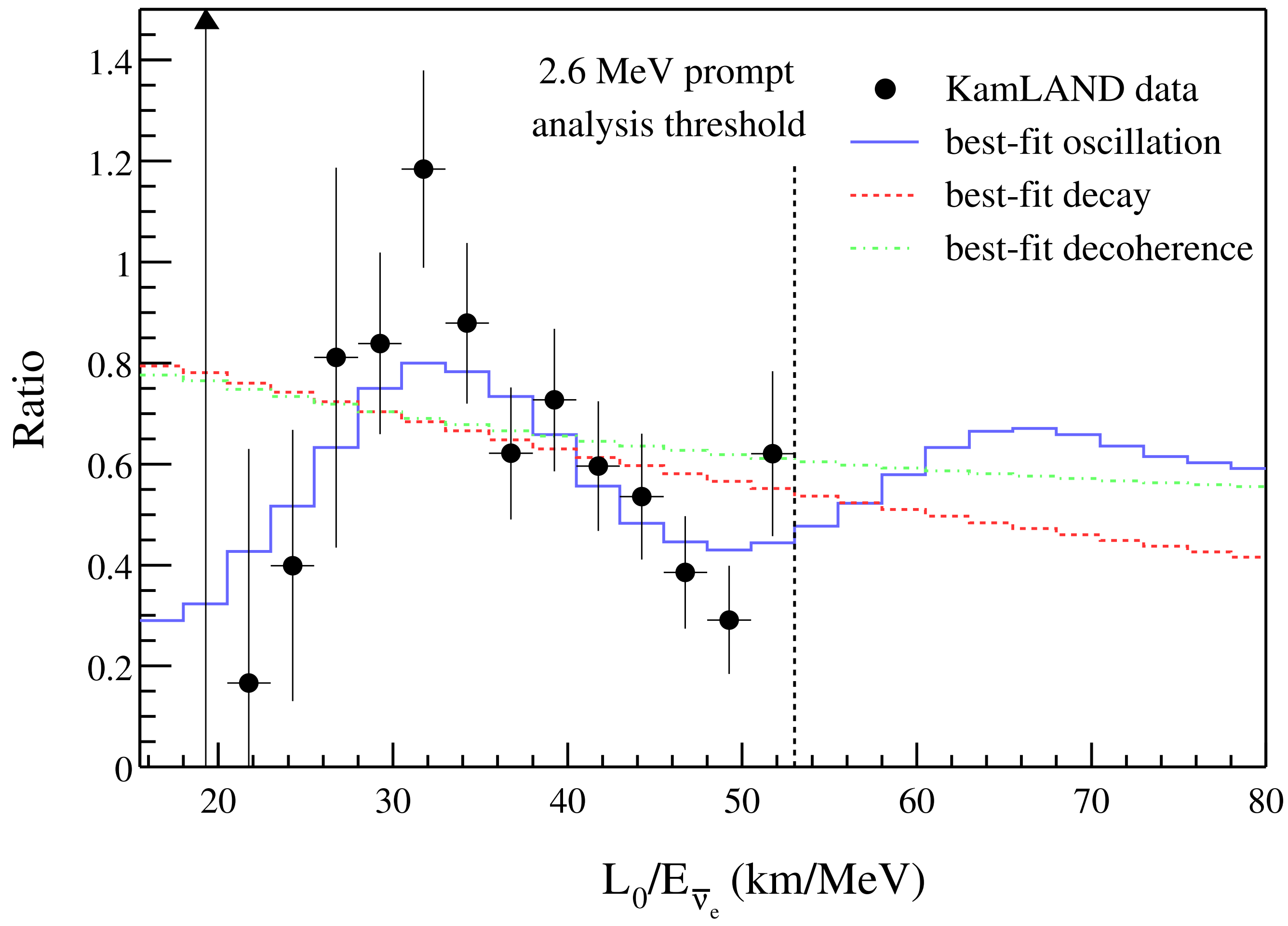

Fig. 5. **Ratio of the observed antineutrino spectrum to the expectation for no-oscillation versus *L/E*. The curves show the expectation for the best-fit oscillation, best-fit decay and best-fit decoherence models taking into account the individual time-dependent flux variations of all reactors and detector effects. The data points and models are plotted**

with L=180 km, as if all antineutrinos detected in KamLAND were due to a single reactor at this distance.

The minimal scheme of three neutrino mixing,

$$\nu_\ell = \sum_{j=1}^{3} U_{\ell j}\nu_j; \quad m_1 < m_2 < m_3,$$

provides two independent Δm^2 and allow to describe solar and atmospheric neutrino oscillation data. There are two possibilities for hierarchy of neutrino mass-squared differences:

H1: $\Delta m^2_{21} \simeq \Delta m^2_{\text{sol}} \ll \Delta m^2_{32} \simeq \Delta m^2_{\text{atm}}$;

H2: $\Delta m^2_{32} \simeq \Delta m^2_{\text{sol}} \ll \Delta m^2_{21} \simeq \Delta m^2_{\text{atm}}$.

For H1, the short oscillations are operating but the long ones have not developed:

$$\Delta m^2_{32} L/E \gtrsim 1, \quad \Delta m^2_{21} L/E \ll 1,$$

$$P\left(\nu_\alpha \to \nu_\beta\right) \simeq 4\left|U_{\alpha 3}\right|^2 \left|U_{\beta 3}\right|^2 \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right),$$

$$P\left(\nu_\alpha \to \nu_\alpha\right) \simeq 1 - 4\left|U_{\alpha 3}\right|^2 \left(1 - \left|U_{\alpha 3}\right|^2\right) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right),$$

$$U^+U = I \to \sum_\alpha \left|U_{\alpha 3}\right|^2 = 1.$$

(For H2: $\Delta m_{32}^2 \to \Delta m_{21}^2$, $\left|U_{\alpha 3}\right|^2 \to \left|U_{\alpha 1}\right|^2$.)

For very long baseline experiments,

$$\frac{\Delta m_{32}^2}{2E} L \gg 1,$$

the averaged (over fast oscillations) survival probability

$$P\left(\nu_e \to \nu_e\right) = \left|U_{e3}\right|^4 + \left[\left(1 - \left|U_{e3}\right|^2\right)^2 - 4\left|U_{e1}\right|^2 \left|U_{e2}\right|^2 \sin^2\left(\Delta m_{21}^2 \frac{L}{2E}\right)\right].$$

The standard parameterization of the Pontecorvo–Maki–Nakagawa–Sakata mixing matrix:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix},$$

where $s_{jk} = \sin\theta_{jk}, c_{jk} = \cos\theta_{jk}; \theta_{12}, \theta_{13}, \theta_{23}$ are the mixing angles; δ is the *CP* violating Dirac phase; α_1, α_2 are Majorana phases.

The Majorana phases are common to an entire column of the mixing matrix, and therefore they have no effect on neutrino oscillations.

In this parametrization (omitting the Majorana phases),

$$U_{e1} = \cos\theta_{12}\cos\theta_{13}, U_{e2} = \sin\theta_{12}\cos\theta_{13}, U_{e3} = \sin\theta_{13}e^{-i\delta};$$

$$U_{\mu 3} = \cos\theta_{13}\sin\theta_{23}, U_{\tau 3} = \cos\theta_{13}\cos\theta_{23}.$$

From experimental data,

$$|U_{e3}|^2 = \sin^2\theta_{13} < 5\times 10^{-2} (\simeq 0).$$

In the leading approximation, neutrino oscillations in atmospheric and solar ranges of Δm^2 are described by two-neutrino formulas with

$$\Delta m^2_{\mathrm{atm}} \simeq \Delta m^2_{32}, \sin^2\theta_{\mathrm{atm}} \simeq \sin^2\theta_{23} \text{ and } \Delta m^2_{\mathrm{sol}} \simeq \Delta m^2_{21}, \sin^2\theta_{\mathrm{sol}} \simeq \sin^2\theta_{12},$$

respectively.

At 99.73% C.L. (3^{1}) the *effective two-neutrino mixing parameters* are constrained in the ranges:

$$1.4\times10^{-3}\ \mathrm{eV}^2 < \Delta m^2_{\mathrm{atm}} < 5.1\times10^{-3}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{\mathrm{atm}} > 0.86;$$

$$5.4\times10^{-5}\ \mathrm{eV}^2 < \Delta m^2_{\mathrm{sol}} < 5.4\times10^{-5}\ \mathrm{eV}^2, \quad 0.30 < \tan^2\theta_{\mathrm{sol}} < 0.64;$$

the allowed ranges for the elements of the mixing matrix:

$$|U| = \begin{pmatrix} 0.76\div0.88 & 0.47\div0.62 & 0.00\div0.22 \\ 0.09\div0.62 & 0.29\div0.79 & 0.55\div0.85 \\ 0.11\div0.62 & 0.32\div0.80 & 0.51\div0.83 \end{pmatrix}.$$

The best fit:

$$\Delta m^2_{\rm atm} = 2.6\times10^{-3}\ {\rm eV}^2, \quad \sin^2 2\theta_{\rm atm} = 1;$$
$$\Delta m^2_{\rm sol} = 6.9\times10^{-5}\ {\rm eV}^2, \quad \tan^2\theta_{\rm sol} = 0.43;$$

$$U_{\rm bf} = \begin{pmatrix} 0.84 & 0.55 & 0.00 \\ -0.39 & 0.59 & 0.71 \\ 0.39 & -0.59 & 0.71 \end{pmatrix}.$$

The neutrino mixing matrix (with all elements large except U_{e3}⁀ is very different from the quark mixing matrix, in which mixing is very small.

Limits on the neutrino masses

● From tritium beta decay $^3H \to {}^3He + e^- + \bar{\nu}_e$ (fitting the shape of the beta spectrum):

$$m_\beta = \left(\sum_k |U_{ek}|^2 m_k^2\right)^{1/2} < 2.05(2.2)\ \text{eV [Troitsk (Mainz), 95\% CL].}$$

It implies an upper limit on the minimal neutrino mass ($\sum_k |U_{ek}|^2 = 1$):

$$m_1 \le m_\beta .$$

- **From the oscillations of atmospheric neutrinos (assuming $0 \le m_1 < m_2 < m_3$):**

$$m_3 > \left(\Delta m^2_{\text{atm}}\right)^{1/2} \simeq 5\times 10^{-2}\ \text{eV}.$$

- **From recent cosmological data (the high-precision CMBR data of the WMAP satellite combined with other astronomical data and some cosmological assumptions):**

$$\sum_k m_k < 0.71\ \text{eV (at } 95\%\ \text{CL)},$$

where the sum runs over all the light ($m_k <\sim 1\ \text{MeV}$) neutrino mass eigenstates that were in thermal equilibrium in the early Universe. For just three light ν_k, taking into account $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}} \ll 1\ \text{eV}^2$, it implies

$$m_k < 0.23\ \text{eV}.$$

Then for the heaviest ν_3,

$$0.05\ \text{eV} < m_3 < 0.23\ \text{eV}.$$

- **From neutrinoless double beta decay $(A,Z) \to (A,Z+2) + e^- + e^-$, the effective Majorana mass**

$$\left|\langle m_{ee} \rangle\right| = \left|\sum_k U_{ek}^2 m_k\right| < 0.3 \div 1.3\ \text{eV}\ \ \text{(Heidelberg–Moscow, IGEX).}$$

[The large uncertainty is due to the difficulty of calculating the nuclear matrix element of the decay].

The nature of neutrino masses: Dirac or Majorana?

To be Dirac or Majorana? That is one of the main unsolved questions of particle physics.

The neutrino oscillations do not probe the nature of the mass.

The Dirac neutrino carries the lepton number that distinguishes it from the antineutrino, and the Dirac neutrino mass is generated just like the quark and charged lepton masses via the standard Higgs mechanism.

The Dirac mass term

$$L^D = -m_D \bar{\nu}\nu = -m_D(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R);$$

$$\nu_{L,R} = \frac{1}{2}(1 \mp \gamma^5)\nu.$$

Here $m_D = y\text{v}/\sqrt{2}$, y is the Yukawa coupling and ⎽ is the vacuum expectation value of the Higgs field. The spinor $\nu = \nu_L + \nu_R$ has 4 independent components.

The Majorana mass terms

$$L_L^M = -\frac{1}{2} m_L(\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c), \quad L_R^M = -\frac{1}{2} m_R(\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c).$$

Here the charged conjugated spinor is defined as follows:

$$\psi^c = C\overline{\psi}^T = C\gamma^{0T}\psi^*, \quad C = i\gamma^2\gamma^0,$$
$$C\gamma^{\mu T}C^{-1} = -\gamma^\mu, \quad C^+ = C^{-1}, \quad C^T = -C.$$
$$C\gamma^{5T}C^{-1} = \gamma^5$$

Since $C\gamma^{5T}C^{-1} = \gamma^5, \quad \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0,$

$$\psi_L^c \equiv (\psi_L)^c = \frac{1}{2}(1+\gamma^5)\psi^c = (\psi^c)_R,$$
$$\psi_R^c \equiv (\psi_R)^c = \frac{1}{2}(1-\gamma^5)\psi^c = (\psi^c)_L.$$

The Majorana mass term violates lepton number by two units, $\Delta L = \pm 2$.
The Majorana neutrino is a true neutral particle identical to its antiparticle:

$$\nu = \nu^c \quad \text{(E. Majorana, 1937).}$$

For the Majorana field,

$$\psi = \psi_L + \psi_R = \psi_L^c + \psi_R^c = \psi_L + \psi_L^c = \psi^c,$$

since $\psi_L^c = \left(\psi^c\right)_R = \psi_R$. This field depends only on the two independent components of ψ_L.

The total Dirac-Majorana mass term

$$L^{D+M} = L^D + L_L^M + L_R^M = -\frac{1}{2}(\overline{\nu}_L^c \overline{\nu}_R)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.},$$

where ν_L and ν_R are independent.

The D-M mass term in the matrix form

$$L^{D+M} = -\frac{1}{2}\bar{N}_L^c M N_L + \text{H.c.},$$

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \quad M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}.$$

Diagonalization gives the mass eigenstates:

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad U^+ U = I, \quad N_L = U \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix},$$

where $m_k \geq 0$.

For the simplest case of a *real* mass matrix M **:**

$$U = O\rho = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}, \quad |\rho_k|^2 = 1;$$

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} \rho_1^2 m_1' & 0 \\ 0 & \rho_2^2 m_2' \end{pmatrix},$$

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}, \quad m_{1,2}' = \frac{1}{2}\left[m_L + m_R \mp \sqrt{(m_L - m_R)^2 + 4m_D^2} \right].$$

Here $\rho_2^2 = 1$ always, and $\rho_1^2 = 1\,(-1)$ if $m_1' \geq 0\,(<0)$.

The diagonalized Dirac-Majorana mass term:

$$L^{D+M} = -\frac{1}{2}\sum_k m_k\,(\bar{\nu}_{kL}^c \nu_{kL} + \bar{\nu}_{kL}\nu_{kL}^c) = -\frac{1}{2}\sum_k m_k \bar{\nu}_k \nu_k\,,$$

$$\nu_k = \nu_{kL} + \nu_{kL}^c = \nu_k^c\,.$$

Therefore, the two massive neutrinos are Majorana particles.

The "seesaw" mechanism for neutrino masses

From experimental data we know that

$$m_\nu \ll m_\ell\,, m_q\,.$$

In order to suppress neutrino masses let us assume that beyond the SM (at ultra-high energy) there exists a mechanism generating the right-handed

Majorana mass term ($m_R \gg m_D$), and the Dirac mass term is generated with the standard Higgs mechanism. The seesaw mechanism [M. Gell-Mann, P. Ramon, R. Slansky (1979), T. Yanagida (1979); R. N. Mohapatra and G. Senjanovic (1980)] is based on the Dirac-Majorana mass matrix with $m_L = 0$ (it is assumed no Higgs triplets):

$$M = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}.$$

Here

$$m_D \sim m_\ell \text{ or } m_q.$$

The neutrino ν_R is completely neutral under the Standard-model gauge group $SU(2)_L \times U(1)_Y$, and m_R is not connected with the SM symmetry breaking scale

$$\mathrm{v} = \left(\sqrt{2}G_F\right)^{-1/2} \simeq 246\ \mathrm{GeV},$$

but is associated to a different higher mass scale, e.g., the GUT-scale:

$$m_R \sim M_{GUT} \gg m_D.$$

Diagonalization of the mass matrix M gives

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad m_1 \simeq \frac{m_D^2}{m_R} \ll m_D, \quad m_2 \simeq m_R \gg m_D;$$

$$\nu_L = i\nu_{1L} \cos\theta + \nu_{2L} \sin\theta,$$

$$\nu_R^c = -i\nu_{1L} \sin\theta + \nu_{2L} \cos\theta,$$

$$\tan\theta = 2m_D / m_R \ll 1.$$

In the case of three generations,

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \nu_R^c = \begin{pmatrix} \nu_{s1R}^c \\ \nu_{s2R}^c \\ \nu_{s3R}^c \end{pmatrix}, \quad M = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix},$$

the seesaw mechanism leads to a mass spectrum of Majorana neutrinos $\{m_k\}$ with 3 light masses ($k = 1,2,3$) and 3 very heavy ones ($k = 4,5,6$):

$$V^T M V = \mathrm{diag}(m_1, \ldots, m_6);$$

$$L^{D+M} = -\sum_{k=1}^{6} \frac{m_k}{2} \bar{\nu}_k \nu_k .$$

$$V = WZ, \quad W^T M W = \begin{pmatrix} M_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix} + \mathrm{O}(M_R^{-1} M_D),$$

$$M_{\text{light}} = -M_D^T M_R^{-1} M_D, \quad M_{\text{heavy}} = M_R .$$

The mixing relation

$$N_L = V n_L, \quad n_L^T = (\nu_{1L}, \ldots, \nu_{6L});$$

$$\nu_{\ell L} = \sum_{k=1}^{6} V_{\ell k} \nu_{kL} \; (\ell = e, \mu, \tau), \quad \nu_{sR}^c = \sum_{k=1}^{6} V_{sk} \nu_{kL},$$

where $|V_{\ell k}|$ $(k = 4,5,6)$ **and** $|V_{sk}|$ $(k = 1,2,3)$ **are very small** $(\sim m_D/m_R)$.

The charged current Lagrangian in terms of the massive neutrino fields ν_k :

$$L_{CC} = -\frac{g}{\sqrt{2}} W_{\alpha}^{+} \sum_{\ell=e,\mu,\tau} \sum_{k=1}^{6} \overline{\ell}_L \gamma^{\alpha} U_{\ell k} \nu_{kL} + \text{H.c.}$$

Here U is the 3×6 neutrino mixing matrix:

$$U = A_L^{+} V,$$

and A_L is the 3×3 unitary matrix arising from diagonalization of the Dirac mass term for the charged leptons:

$$L_{ml} = -\overline{l}_R^{0} M_l l_L^{0} + \text{H.c.} = -\overline{l}_R D_l l_L + \text{H.c.} = -\sum_{\ell=e,\mu,\tau} m_{\ell} \overline{\ell}\ell,$$

$$l^T = (e,\mu,\tau),\ \ell = \ell_L + \ell_R;$$

$$A_R^{+} M_l A_L = D_l = \text{diag}\left(m_e, m_{\mu}, m_{\tau}\right),$$

$$l_P^0 = A_P l_P\ (P = L,R),\ A_P^{+} A_P = I,$$

where l^0 and l are weak and mass eigenstates, respectively.

In general case the number n_s of electroweak-singlet ("sterile") neutrinos ν_{sR} is arbitrary, and the seesaw mechanism yields a set of 3 light masses and n_s

large masses. There exists a large number of seesaw models in which both m_D and m_R vary over many orders of magnitude, with m_R ranging somewhere between the TeV scale and the GUT scale ($\sim 10^{15} \div 10^{16}$ GeV).
For example, from the atmospheric neutrino oscillations,

$$m_3 > \sqrt{\Delta m^2_{\rm atm}} \equiv m_a \simeq 5\times 10^{-2}\ \text{eV}.$$

Assuming

$$m_a = \frac{m_D^2}{m_R},$$

we obtain

$$m_R = \frac{m_D^2}{m_a} \simeq 0.6\times 10^{15}\ \text{GeV} \quad \text{for}\ \ m_D = m_t \simeq 174\,\text{GeV},$$

and

$$m_R \simeq 5\,\text{TeV} \quad \text{for}\ \ m_D = m_e \simeq 0.511\,\text{MeV}.$$

Consider a possible scenario of the generation of the Dirac-Majorana mass term suitable for the seesaw mechanism. The grand unified group $SO(10)$ can break to the SM group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ through the chain

$$SO(10) \xrightarrow{\Lambda_{GUT}} G_{SM} \times U(1)_{B-L} \xrightarrow{\mathrm{V}} G_{SM} \xrightarrow{\mathrm{v}} SU(3)_c \times U(1)_{em},$$

inducing the mass term

$$L^{D+M} = -\bar{\nu}_L M_D \nu_R - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R + \text{H.c.},$$

where M_D is a $3\times n_s$ Dirac mass matrix and M_R is a $n_s \times n_s$ Majorana mass matrix:

$$M_D = y\mathrm{v}/\sqrt{2}, \quad M_R = Y\mathrm{V}/\sqrt{2},$$

y and Y are matrices of Yukawa couplings. The breaking scales:

$$\Lambda_{GUT} \sim 10^{15} \div 10^{16}\ \text{GeV},\ \mathrm{V} \sim 1 \div 10\ \text{TeV},\ \mathrm{v} = \left(\sqrt{2}G_F\right)^{-1/2} \simeq 246\ \text{GeV}.$$

Diagonalization of the neutrino mass matrix by means of a unitary $(3+n_s)\times(3+n_s)$ matrix V gives 3 light and n_s heavy Majorana neutrinos:

$$\nu_{\ell L} = \sum_{k=1}^{3} V_{\ell k} \nu_{kL}^{\text{light}} + \sum_{k=4}^{n_s+3} V_{\ell k} \nu_{kL}^{\text{heavy}}.$$

Conclusion

Neutrino physics is a booming field of research involved with particle physics, nuclear physics, astrophysics, and cosmology. In general, neutrinos play a special role in physics due to their intimate connection with fundamental symmetries and conservation laws:

- **energy and momentum,**
- **lepton number,**
- **parity and charge conjugation,**
- ***CP* invariance,**
- ***CPT* and Lorentz invariance.**

The recent discovery of neutrino oscillations has opened physics of massive and mixed neutrinos. There is a general consensus that neutrino masses have their origin in a New Physics beyond the SM.

The deeper understanding of the neutrino properties remains an open fundamental problem.