Higher dimensional perspectives on N=2 black holes

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Black holes in d=4, N=2 supergravity

Type II string theory compactified on Calabi-Yau manifolds is described at low energies by N=2 d=4 supergravity. This description is valid at distance scales larger than the size of the Calabi-Yau.

In type IIA supergravity we will look at charged black holes. They are charged with respect to $h^{(1,1)}+1$ U(1) gauge fields. There are scalar partners of $h^{(1,1)}$ of the gauge fields, they parameterize the Kahler moduli space of the Calabi-Yau.

Supersymmetric black holes are extremal Reisner-Nordstrom type. Attractor mechanism operates (Ferrara, Kallosh, Strominger '95) - Black holes interpolate between $R^{3,1}$ asymptotically and $AdS_2 \times S^2$ in the near-horizon. The near-horizon geometry is independent of the asymptotic values of the scalar fields only their values at the horizon which are determined by the charges.

Area of the horizon is given by $4\pi R^2 = 4\pi(q_0 c_{ijk} p^i p^j p^k)^{1/2}$ where q and p are electric and magnetic charges with respect to the U(1) fields.

11-dimensional origins

Our starting point is M-theory (aka 11-dimensional supergravity) compactified on a Calabi-Yau manifold. I.e. space-time is 5-dimensional Minkowski space times a Calabi-Yau manifold (a complex Ricci flat manifold of SU(3) holonomy.

We introduce now an M5-brane wrapping a 4-cycle in the Calabi-Yau. The supergravity solution has some well studied properties.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

$$
\begin{aligned}
F &= -\frac{i}{2}r^2\partial_r g_{m\bar{n}}dz^m \wedge dz^{\bar{n}} \wedge dvol(S^2) + \frac{i}{2}r^2\partial_m H dz^m \wedge dr \wedge dvol(S^2) \\
&- \frac{i}{2}r^2\partial_{\bar{m}} H dz^{\bar{m}} \wedge dr \wedge dvol(S^2)
\end{aligned}
$$

(H. Cho, M. Emam, D. Kastor, J. Traschen hep-th/0009062, T.Z. Husain hep-th/03020

• The metric $g_{m\bar{n}}$ is Kahler in z^m but also depends on the transverse Directions x^i as well. It satisfies the equation:

$$\nabla_\perp^2 g_{m\bar{n}} + 2\partial_m \partial_{\bar{n}} H = 0$$

Where

$$det(g) = aH^2|h|^2$$

Thus $g_{m\bar{n}}$ satisfies a non-linear partial differential equation which depends on the 4-cycle the M5-brane wraps.

• The metric describes a string (along the y-coordinate) transverse to the six dimensional space that was once the Calabi-Yau manifold.
•The solution is translationally invariant in this direction and we can further compactify the y-direction to a circle and arrive at a point like object in four dimensions traverse to the Calabi-Yau and circle.
•Notice that the metric is not a product space since H ar$g_{m\bar{n}}$both depend on z^m and x^i.
•The metric g is not Calabi-Yau anymore although it is still Kahler.
•This black hole has zero horizon area (Maldacena, Strominger, Witten '97)

We can add momentum to the above solution in the compact y-direction so as to not break any further supersymmetry:

$$ds^2 = H^{-1/3}(-f^{-1}dt^2+f(dy+(f^{-1}-1)dt)^2)+H^{2/3}\sum_{i=1}^{3}(dx^i)^2+2H^{-1/3}g_{m\bar{n}}dz^m dz^{\bar{n}}$$

Where f satisfies the differential equation

$$H^{-1}\nabla_{\perp}^2 f + 2g^{m\bar{n}}\partial_m\partial_{\bar{n}}f = 0$$

Along with the equation satisfied by g completely specifies the black hole geometry.

The full black hole geometry is complicate and inherently 11-dimensional - I.e. it is not a product geometry or even conformal to a product.

Type IIA description

The above solution for a small radius of the circle along y has a 10-dimensional description in terms of D4-branes wrapped on 4-cycles and D0-branes.

$$
\begin{aligned}
ds_{10}^2 &= -H^{-1/2}f^{-1/2}dt^2 + H^{1/2}f^{1/2}(dr^2 + r^2 d\Omega^2) + 2H^{-1/2}f^{1/2}g_{m\bar{n}}dz^m dz^{\bar{n}} \\
C_1 &= Adt = (f^{-1} - 1)dt \\
e^{\phi} &= H^{-1/4}f^{3/4} \qquad (12) \\
dC_3 &= F
\end{aligned}
$$

C_1 couples electrically to D0-branes and C_3 couples magnetically to D4-branes.

This geometry is, again, inherently 10-dimensional.

Some generalities about black hole geometries

•Far from the black hole we should get back the vacuum space-time: $R^{3.1} \times CY_3$

•The black hole geometry interpolates between the vacuum and the (yet to be determined) near-horizon regime.

• Normally the near horizon regime is thought of as a truncation of the full space-time to the region close to the horizon.

There are some characteristics common to all known examples of extremal near-horizon supergravity solutions (Vafa '00): the localized sources (branes) for the gravitational and matter fields appear to smooth out as one probes them at distances of the order of the Planck scale.

In fact, in the near horizon limit: localized sources $\rightarrow$ non-singular flux

The flux is through finite sized cycles and the geometry is smooth.
(Key example: D3-brane geometry $\rightarrow$ $AdS_5 \times S^5$, $F_5 = R(vol(AdS_5) + vol(S^5))$)

Near Horizon geometry

We make the following ansatz for the form fields:

$$\begin{aligned} F &\rightarrow F' = \omega_2 \wedge dvol(S^2) \\ G_8 &\rightarrow G_8' = \omega_6 \wedge dvol(S^2) \end{aligned}$$

Where G_8=*dC_1, and ω_2, ω_6 are 2 and 6 forms independent of r and dvol(S^2) is the volume form for a unit sphere in the transverse space.

These forms are non-singular and represent flux, when integrated over appropriate cycles they calculate the number of branes that were present.

Comparing with our previous expressions we find that this ansatz result

$$ds_{10}^2 = (-\frac{r^2}{R^2}dt^2 + \frac{R^2}{r^2}dr^2) + R^2 d\Omega_2^2 + 2h_{mn}dz^m dz^n$$

Where R is a constant and h is a Calabi-Yau metric.

The space-time is $AdS_2 \times S^2 \times CY_3$ and the remaining fields can be expressed as:

$$
\begin{aligned}
F' &= \omega_2 \wedge dvol(S^2) = R\Omega^3 J_h \wedge dvol(S^2) \\
G_2 &= dC_1 = d(f^{-1} - 1) = \Omega^3 R^{-1} dt \wedge dr \\
e^{-\phi'} &= \Omega^3
\end{aligned}
$$

The eleven dimensional geometry is:

$$
\begin{aligned}
ds_{11}^2 &= \Omega^2\left(-\frac{r^2}{R^2}dt^2 + \frac{R^2}{r^2}dr^2\right) + \Omega^{-4}\left(dy + \left(\frac{r}{a_0} - 1\right)dt\right)^2 \\
&+ \Omega^2 R^2 d\Omega_2^2 + 2k_{mn} dz^m dz^n
\end{aligned}
$$

Here $k=\Omega^2 h$ is also a Calabi-Yau metric. The geometry is in fact $AdS_3 \times S^2 \times CY_3$.

Some comments

• We argued that the black hole geometry interpolates between the vacuum $R^{3,1}\times CY_3$ and $AdS_2\times S^2\times CY_3$. The vacuum and near-horizon CY_3 are in general different Calabi-Yau manifolds.

• The full black hole geometry is very complicated and not factorizable but in the two extremes it takes a fairly simple form as a product.

• This solution is consistent with the idea of attractors. The metric of the near-horizon Calabi-Yau and the form fields are directly related to each other and determined completely in terms of the charges. While the asymptotic Calabi-Yau does not affect the near horizon geometry.

•If the four form carries flux through a 4-cycle $\Sigma_2\times S^2$ then its easy to show that the size of the cycle is bounded from below by the charge of the black hole:

$$\int_{\Sigma_2\times S^2} F_4 = R^{-1}\Omega^3 \int_{\Sigma_2} J_h \leq R^{-1}\Omega^3 V_{\Sigma_2}$$

Generalizing to other branes and fluxes

We have seen that in the D4/D0 brane case the ten dimensional near-horizon geometry is $AdS_2 \times S^2 \times CY_3$. We might imagine that in general (when D0/D2/D4/D6 branes are present) the 11-dimensional geometry is a U(1) bundle over $AdS_2 \times S^2$ times a complex manifold.

$$ds^2 = R_1^2(-r^2 dt^2 + \frac{dr^2}{r^2}) + R_2^2(d\theta^2 + sin^2\theta d\phi^2) + R_3^2(dy + A_i dx^i)^2 + 2g_{mn}dz^m dz^{\bar{n}}.$$

If we also assume that all the fields respect the isometries of $AdS_2 \times S^2$ then we are led to the ansatz:

$$\begin{aligned} F_4 &= c_1 J \wedge \eta + c_2 J \wedge J + c_3 dt \wedge dr \wedge \eta + c_4 dt \wedge dr \wedge J \\ G_2 &= a dt \wedge dr + b\eta \end{aligned}$$

Where a,b, c_i are constants and η is the volume form on a unit S^2. The equations of motion

$$R_{ab} = \frac{1}{12} F_{acde} F_b{}^{cde} - \frac{1}{144} G_{ab} F_{cdef} F^{cdef}$$

relate the coefficients a, b, c_i, R_i to each other.

Solving the equations of motion give us some constraints on M. We find that

- M is Kahler
- M is Einstein with a cosmological constant which is either 0 or positive

One interesting case is that of the D6 wrapping the Calabi-Yau and D2-branes wrapping a 2-cycle in the Calabi-Yau. In that case the solution can be written very simply as:

$$\begin{aligned} ds^2 &= R_1^2(-r^2dt^2 + \frac{dr^2}{r^2}) + R_1^2(d\theta^2 + \sin^2\theta d\phi^2) \\ &+ R_3^2(dy + R_3^{-1}R_1\cos\theta d\phi)^2 + 2g_{mn}dz^m dz^n \\ F_4 &= R_1 dt\wedge dr\wedge J \\ G_2 &= R_3^{-1}R_1\sin\theta d\phi\wedge d\theta \end{aligned}$$

The geometry here is $AdS_2\times S^3\times CY_3$.

Conclusions and outlook

• The geometry of wrapped branes is complicated but simplifies considerably in the two limiting regimes - the asymptotic and near-horizon regions.
• There is still much to be learned about general near-horizon geometries of N=2 black holes.
• Some interesting directions involve generalizing Sen's considerations concerning the entropy of black holes with near-horizon geometry of $AdS_2 \times S^p$ to ones considered here.