Alfven wave heating of solar wind protons using a generalized non maxwellian distribution function

by

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- Non maxwelian distribution functions
- Landau damping of oblique Alfven waves
- Observed cooling rate
- CGL cooling rate
- Power requirements
- Comparison between observation and theoretical model

Non maxwelian distribution functions

- Data from real (space) shows distribution functions deviate from maxwellians
- Presence of high energy tails, shoulders in the profile, peaks or flat tops
- The default assumptions of using maxwellians seems no longer valid
- Generally space plasmas are turbulent thus thermodynamic equilibrium does not exist

- Quasi thermodynamic equilibrium state valid for turbulent systems
- Hasegawa et.al. showed how a non maxwellian distribution can emerge as a natural consequence of super thermal radiation fields in plasmas
- Entropy generalization using non extensive statistics

$$f_{(r,q)0}=\frac{3(q-1)^{\frac{-3}{2(1+r)}}\Gamma(q)}{4\pi\Psi^{2}{}_{\perp}\Psi_{\parallel}\,\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\left(1+\frac{1}{q-1}\left(\left(\frac{v_{\parallel}}{\Psi_{\parallel}}\right)^{2}+\left(\frac{v_{\perp}}{\Psi_{\perp}}\right)^{2}\right)^{r+1}\right)^{-q}$$

$$\Psi_{\parallel,\perp}=v_{t\parallel,\perp}\sqrt{\frac{3(q-1)^{\frac{-1}{(1+r)}}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}}$$

$$q(r+1)>5/2$$

- Generalized Lorentzian or κ distribution $r \to 0$

$$f_{\kappa 0} = \left(1 + \frac{1}{\kappa}\left(\left(\frac{v_{\parallel}}{\Theta_{\parallel}}\right)^2 + \left(\frac{v_{\perp}}{\Theta_{\perp}}\right)^2\right)\right)^{-(\kappa+1)} \frac{3(\kappa)^{\frac{-3}{2}}\Gamma(\kappa+1)}{4\pi\Theta^2{}_{\perp}\Theta_{\parallel}\,\Gamma\left(\kappa - \frac{1}{2}\right)\Gamma\left(\frac{5}{2}\right)}$$

- Davydov Druyvestien $q \to \infty$

$$f_{DD} \propto \exp(-(v^2/v_t^2)^{r+1})$$

- Maxwellian $r \to 0, q \to \infty$
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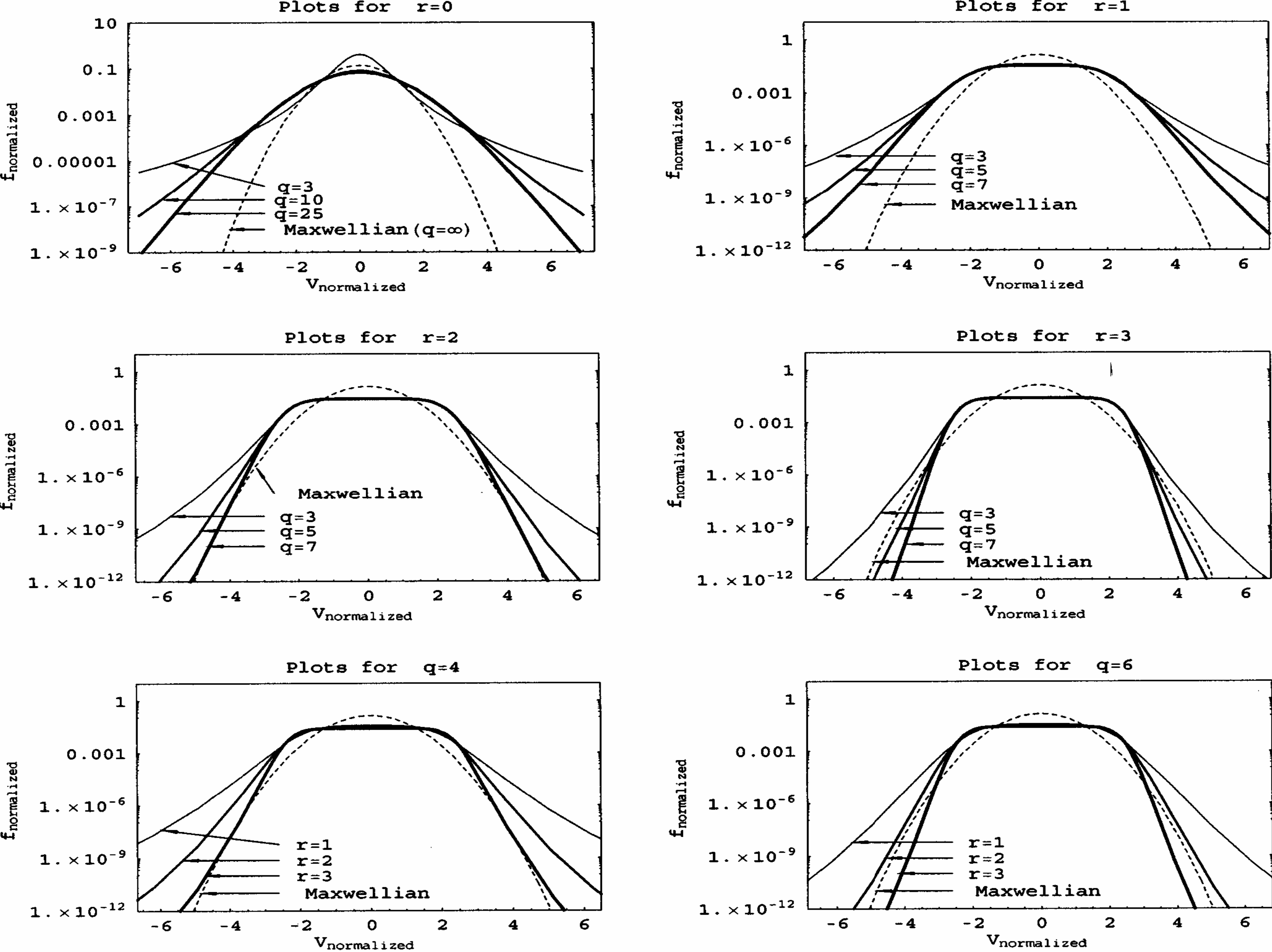
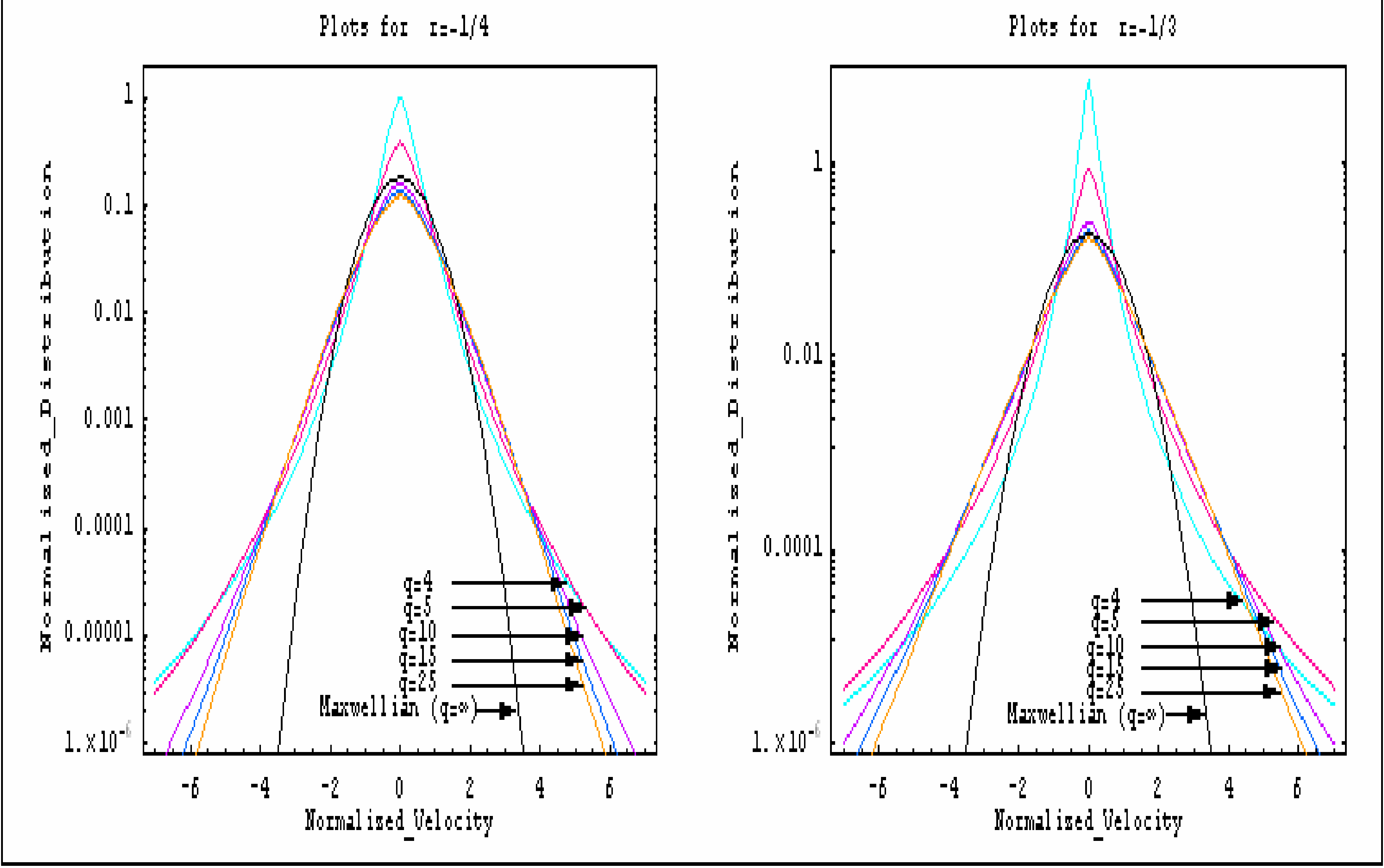


Plots for r=0
Plots for r=1
Plots for r=2
Plots for r=3
Plots for q=4
Plots for q=6
f normalized
V normalized
q=3
q=10
q=25
Maxwellian (q=∞)
q=5
q=7
Maxwellian
r=1
r=2
r=3
10
0.1
0.001
0.00001
1. × 10^-7
1. × 10^-9
1
1. × 10^-6
1. × 10^-12
-6
-4
-2
0
2
4
6



Plots for r=-1/4
Plots for r=-1/3
Normalized_Distribution
Normalized_Velocity
q=4
q=5
q=10
q=15
q=25
Maxwellian (q=∞)
1
0.1
0.01
0.001
0.0001
0.00001
1.x10^-6
-6
-4
-2
0
2
4
6

Solar wind

- Stream of charged particles emanating from the sun
- 95% hydrogen plasma + alpha particles
- Dominated by Alfven waves – fast streams
- Turbulent
- Solar magnetic field is dragged out by the Alfven waves
- In situ obsevations between 0.3 – 1 A.U.
- Protons should cool as the sw expands

- Adiabatic eqn of state $T \propto r^{-\beta}$ $\beta = 4/3$
- Observations show that protons do not cool adiabatically $T \propto r^{-\alpha}, 0.3 \le \alpha \le 4/3$

  for fast and slow streams
- Some local heating mechanism at work for fast speed streams
- Landau damping of oblique Alfven waves

Power dissipated due to Landau damping of Alfven waves

$$\vec{B}_0 = B_0 \hat{z}$$

- We consider small angles of propagation $E_z << E_x$

$$P_J = \frac{1}{16\pi} \sum_j \frac{\varepsilon_j \omega^2{}_{pj}}{\Omega_j} E^* M_j E$$

general expression for power dissipated per unit vol (Stix)

For Landau damping we need only M_{zz}

$$M_{zz} = -\sum_j \frac{\Omega_j \varepsilon_j 2i}{\Psi_{Xj} k_X} \begin{pmatrix} \xi^2 \left(Z^{(r,q)}{}_1(\xi) - \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} Z^{(r,q)}{}_2(\xi) \right) + \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \xi^4 Z^{(r,q)}{}_1(\xi) + \xi A' C_1 \\ -\frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \xi A' B' C_3 + \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \xi A' C_2 + \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \xi^3 A' C_1 \end{pmatrix}$$

$$Z^{(r,q)}{}_1(\xi) = \frac{3(q-1)^{\frac{-3}{2(1+r)}} \Gamma(q)}{4\Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(1 + \frac{3}{2(1+r)}\right)} \int_{-\infty}^{+\infty} \frac{\left(1 + \frac{1}{(q-1)} \left(s^2\right)^{r+1}\right)^{-q}}{(s-\xi)} ds$$

$$Z^{(r,q)}{}_2(\xi) = \frac{3(q-1)^{\frac{-3}{2(1+r)}} \Gamma(q)}{4\Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(1 + \frac{3}{2(1+r)}\right)} \frac{q(1+r)(q-1)^q}{(-1+q+qr)}$$

$$\int_{-\infty}^{+\infty} \frac{\left(s^2\right)^{1-q-qr} 2F1[q+1, q - \frac{1}{1+r}, q + \frac{1}{1+r}, -(q-1)\left(s^2\right)^{(-r-1)}]}{(s-\xi)} ds$$

$$s = \frac{v_X}{\Psi_{Xj}}, \xi = \frac{\omega}{k_X \Psi^2{}_{Xj}}$$

$$P_{(r,q)j} = -\frac{1}{8\pi}\sum_j \frac{\omega^2{}_{pj} i}{\Psi_{\parallel\ j} k_{\parallel\ \}}} \left(\begin{aligned} &\frac{v^2{}_A}{\Psi^2{}_{\parallel\ j}} \left(Z^{(r,q)}{}_1(\xi) - \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} Z^{(r,q)}{}_2(\xi) \right) + \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \frac{v^4{}_A}{\Psi^4{}_{\parallel\ j}} Z^{(r,q)}{}_1(\xi) \\ &+ \frac{v_A}{\Psi_{\parallel\ j}} A'C_1 - \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \frac{v_A}{\Psi_{\parallel\ j}} A'B'C_3 + \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \frac{v_A}{\Psi_{\parallel\ j}} A'C_2 \\ &+ \frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} \frac{v^3{}_A}{\Psi^3{}_{\parallel\ j}} A'C_1 \end{aligned} \right)$$

$$\times \left(\frac{k_{\langle}\ k_\perp}{c\Omega^2{}_p} \times v^3{}_A \frac{m_e}{m_p} B_y \right)^2$$

$$\frac{E_z}{E_x} \cong \frac{k_\perp}{k_{II}} \frac{\omega^2}{\Omega_p^2} \frac{m_e}{m_p}$$

Wu & Huba 1975

$$E_z = \frac{k_\perp}{k_{II}} \frac{\omega^2}{\Omega_p^2} \frac{m_e}{m_p} \frac{\omega}{ck_z} B_y$$

Expanding the plasma dispersion functions for large values of the argument we obtain

$$P_{(r,q)j} = \frac{1}{8}\sum_j \frac{\omega^2{}_{pj}\, i}{\Psi_{\parallel\ j}\left|k_\parallel\right|}\left(\frac{v^2{}_A}{\Psi^2{}_{\parallel\ j}}\right)\left(\begin{array}{l}\left(1+\frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j}\left(\frac{v^2{}_A}{\Psi^2{}_{\parallel\ j}}\right)\right)\left(1+\frac{1}{q-1}\left(\frac{v^2{}_A}{\Psi^2{}_{\parallel\ j}}\right)^{1+r}\right)^{-q} \\ -\frac{k^2{}_\perp \Psi^2{}_{\perp j}}{2\Omega^2{}_j} B'\left(\frac{v^2{}_A}{\Psi^2{}_{\parallel\ j}}\right)^{1-q-qr} \\ \times 2F1[q+1, q-\frac{1}{1+r}, q+\frac{1}{1+r}, -(q-1)\left(\frac{v^2{}_A}{\Psi^2{}_{\parallel\ j}}\right)^{(-r-1)}]\end{array}\right)$$

$$\left(\frac{k_\parallel\ k_\perp}{c\Omega^2{}_p}\times v^3{}_A\,\frac{m_e}{m_p} B_y\right)^2$$

$$B_y^2(\vec{k}) = a_p L_0^3 \langle \delta B_y^2 \rangle (kL_0)^{-p}$$

- Here an isotropic turbent spectrum is taken for the sw (Shah, Iess and Dobrowolny 1986). *L* is the outer scale of the turbulence and *p* is the spectral index of the power law and *a* is a constant
- The total power dissipated per unit volume is

$$R^{alf.}{}_j = \int P_j d\Omega k^2 dk = \int_0^{2\pi} \int_0^{\pi} \int_0^{\frac{1}{r_p}} P_j d\phi Sin\theta d\theta k^2 dk$$

$$B = B_0\left[\frac{\left(x^{-2} + x^{-4}\right)}{2}\right]^{\frac{1}{2}}, T_{\mathrm{X}\ j} = T_{\mathrm{X}\ j0}\, x^{-\alpha},$$

$$r_p = \left[x^{1-\frac{\alpha}{2}}\right]\left[\frac{\left(x^{-2} + 1\right)}{2}\right]^{\frac{-1}{2}}, \Omega_p = \Omega_{p0}\left[\frac{\left(x^{-2} + x^{-4}\right)}{2}\right]^{\frac{1}{2}},$$

$$\omega^2{}_{pj} = \omega^2{}_{pj0}\, x^{-2}, v_A = \frac{v_A}{x^{-1}}\left[\frac{\left(x^{-2} + x^{-4}\right)}{2}\right]^{\frac{1}{2}},$$

$$\left\langle \delta B^2{}_y \right\rangle = \left\langle \delta B^2{}_y \right\rangle B^2{}_0\left[\frac{\left(x^{-2} + x^{-4}\right)}{2}\right]$$

$$R^{alf}{}_{(r,q)j} = \frac{\pi}{8} A' \sum_j \frac{\omega^2{}_{pj0}}{v_{A0}} \left(\frac{v_{A0}}{c}\right)^2 \left(\frac{v_{A0}}{\Psi_{\parallel\ j0}}\right)^3 \left(\frac{v_{A0}}{\Omega_{p0}}\right)^4 \left(\frac{m_e}{m_p}\right)^2 \frac{a_p}{(6-p)} \left(\frac{L_0}{r_{p0}}\right)^{6-p} L^{-3}{}_0 \left\langle \delta B^2{}_y \right\rangle_0 B^2{}_0$$

$$\left(\frac{x^{-2}+1}{2}\right)^{6-\frac{p}{2}} x^{-6+p} \tilde{T}^{\left(\frac{p}{2}-\frac{9}{2}\right)} \left[1+\frac{1}{q-1}\left(\left(\frac{x^{-2}+1}{2}\right)\left(\frac{\sqrt{2}v_{A0}}{A'_1\Psi_{\parallel\ j0}}\right)^2 \tilde{T}^{-1}\right)^{1+r}\right]^{-q}$$

$$x = r / r_0$$

$$c = 3\times10^{10}\, cm/\sec, a_p = \frac{p-3}{4}, p = 7/2$$

$$B_0 = 5\times10^{-5}\, G, \left\langle \delta B^2{}_y \right\rangle = 0.25, m_p = 1.672\times10^{-24}\, g,$$

$$m_e = 9.1094\times10^{-28}\, g, L_0 = 3.2\times10^{10}\, cm$$

$$n = 5/cm^3, T_{II} = 1.5\times10^5\, K$$

$$R^{alf}{}_{(r,q)j} = \left(1.8\times10^{-19}\right)A'\left(\frac{1}{A'_1}\right)^{\frac{11}{2}}\left(\frac{x^{-2}+1}{2}\right)^{\frac{17}{4}} x^{\frac{-5}{2}}\tilde{T}^{\frac{-11}{4}}\left[1+\frac{1}{q-1}\left(\frac{2}{A'^2_1}\right)^{1+r}\right]^{-q+q}$$

$$\left[1+\frac{1}{q-1}\left(\left(\frac{x^{-2}+1}{2}\right)\left(\left(\frac{2}{A'^2_1}\right)\right)^2\tilde{T}^{-1}\right)^{1+r}\right]^{-q}$$

$$\tilde{T} = \frac{T_{II}(x)}{T_{II0}}$$

Data analysis

$$\log f_{(r,q)} = a - q\log\left[1+\frac{1}{q-1}\left[cv^2\left(b\cos^2\theta+\sin^2\theta\right)^{(r+1)}\right]\right]$$

$$a=\log\left(\frac{A_1}{\pi A_2^{3/2}}\right), b=\frac{v^2{}_{T\perp}}{v^2{}_{T\text{X}}}, c=\frac{1}{A_2^{\,2}v^2{}_{T\perp}}$$

Day	a	b	c	q	r
21st Feb.2001	-17.4536	0.9	10^{-13}	11.219	-0.7000
22nd Feb.2001	-15.3374	0.7	10^{-12}	05.651	-0.6600
11th Feb.2002	-15.0607	0.8	10^{-12}	05.967	-0.6665

Power requirements of the sw due to adiabatic cooling

- Sw appears to cool less rapidly than expected
- Some additional source of heating
- We *first* estimate the rate at which parallel thermal energy varies on the basis of obs.
- *Secondly* we obtain heating or cooling rate due to a known theoretical law or eqn of state
- The difference between these two will give the power requirement of the sw protons
- This will then be compared power dissipated due to Landau damping of Alfven waves

Variation of parallel energy density according to obs

$$T_{\parallel}^{obs} = T_{\parallel 0}^{obs} x^{-\alpha}, 0.3 < \alpha < 4/3$$

$$R^{obs.} = v_{sw} \frac{d}{dr}(n k_B T_{\parallel}^{obs}) = -v_{sw} n_0 k_B T_{\parallel 0}^{obs} \left(\frac{2+\alpha}{r_0} \right) x^{-3-\alpha}$$

Heating rate due to known eqn of state

- CGL theory gives a good approximation to a collisionless plasma (SW)

$$T^{adb.}{}_{\parallel}\left(\frac{B}{n}\right)^2 = const.$$

$$R^{adb} = \frac{-v_{sw} n_0 k_B T^{obs.}{}_{\parallel 0}}{r_0}\left(\frac{-4}{x\left(1+x^2\right)^2} - \frac{4}{x^3\left(1+x^2\right)}\right)$$

Power requirements of the solar wind

$$R^{req.} = R^{adb.} - R^{obs.}$$

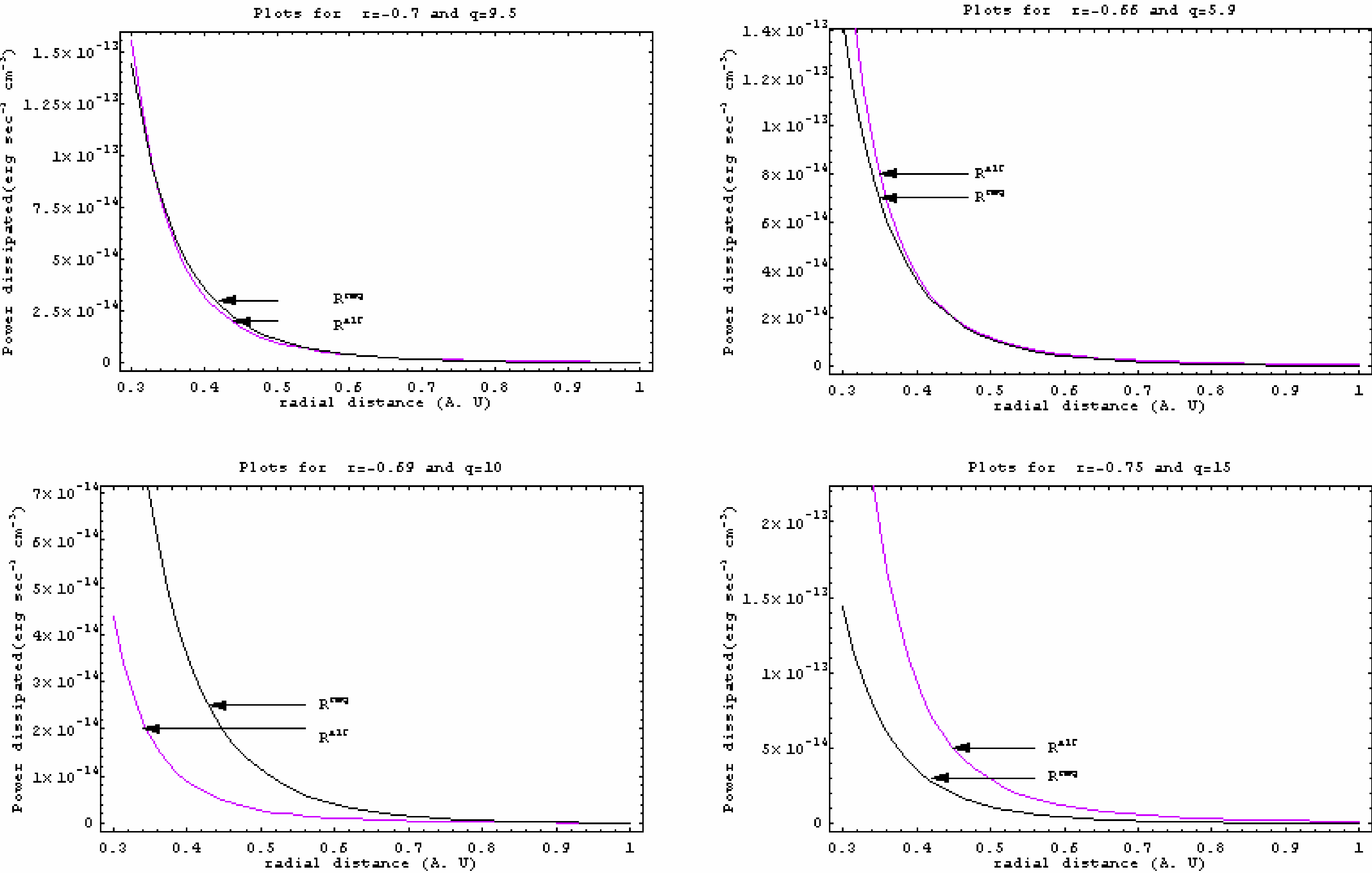
$$\alpha = 1.33, v_{sw.} = 4.5 \times 10^{7}\, cm/\sec,$$

$$R^{req.} = 1.0 \times 10^{-16}\, ergcm^{-3} \sec^{-1}$$

$$r = -0.7, q = 9.5$$

$$R^{alf.} = 4.4 \times 10^{-16}\, ergcm^{-3} \sec^{-1}$$

Radial evolution of power required and due to Landau damping of Alfven waves


Plots for r=-0.7 and q=9.5
Plots for r=-0.66 and q=5.9
Plots for r=-0.69 and q=10
Plots for r=-0.75 and q=15
Power dissipated(erg sec^{-1} cm^{-3})
radial distance (A. U)
R^{air}