THE DECAY OF MASSIVE SCALAR FIELD

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IN ROTATING UNIVERSE

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1. Intoduction

In this study, in the context of general relativity, we have investigated the dynamics of Gödel type rotating universe with massive scalar field, viscous fluid and heat flux in the presence of cosmological constant.

In recent years, there have been a considerable interest in cosmology with scalar fields (zero-mass or massive), as they help in explaining the creation of matter in cosmological theories, represent matter fields with spinless quanta and can be describe gravitational fields[1, 2, 3, 4, 5, 6].

1. Intoduction

The concept of scalar field was introduced by Dirac[7] (in trying to explore the idea of Mach's principle) and then Klauze[8], Brans and Dicke[9] studied the first scalar-tensor theory. Today they find new justification through particles physics theories.

1. Intoduction

Hence the Higgs mechanism explaining the mass of particles is a massive scalar field[10]. Therefore, the study of zero-mass or massive scalar fields in general relativity has drawn the attention of many workers[11, 12, 13, 14].

1. Intoduction

On the other hand, some modern astronomical observations indicate that the universe is rotating[15, 16, 17, 18].

Birch, from CMB observations, found that the magnitude of the cosmic rotation is $w = 10^{-13}$ rad.year^{-1}.

1. Intoduction

For the first time attention to cosmological models with rotation was drawn in 1946 by Gorge Gamov (*Nature*, ***1958***, *549)*, although we should mention also the earlier work of Lanczos in 1924 (*Z. Physik*, ***21***, *73).*

Soon after this, in 1949, Kurt Gödel had suggested to describe cosmic rotation with the help of a space-time metric.

A rotating universe with vanishing expansion and shear appeared with the advent of Gödel's universe[19].

1. Intoduction

As compared to the Gödel’s world, the model suggested earlier by Lanczos appears to be less physical in that it describes a universe as a rigidly rotating dust cylinder of infinite radius.

Dust density in this solution (letter rederived by van Stockum) diverges at radial infinity.

Using the Gödel model one can clearly understand the idea of the cosmic rotation of matter in the universe.

1. Intoduction

In particular, homogeneous, anisotropic rotating cosmologies were analyzed and very strong upper limits on the value of the cosmic rotation were reported[20, 21, …].

1. Introduction

Viscosity plays an important role in explaining many physical features of the homogeneous world models[22, 23, 24].

Homogeneous cosmolocigal models filled with viscous fluid have been widely studied[25, … 33].

2. Formulation of the problem

We consider a non-static, homogeneous and anisotropic Gödel type space-time for rotating universe in the form[13]

$$ds^2 = dx^2 + dz^2 - (dt + He^x dy)^2 + \frac{1}{\alpha^2} H^2 e^{2x} dy^2 \qquad \text{...(1)}$$

- **H** is a function of t
- **α** is a constant

$$G_{ik} \equiv R_{ik} \frac{1}{2} R \ \mathrm{g}_{ik} + \Lambda \ \mathrm{g}_{ik} = -\mathrm{T}_{ik} \qquad \text{...(2)}$$

2. Formulation of the problem

The energy-momentum tensor for cosmic matter distribution including massive scalar field, viscous fluid and heat flux is given in the form

$$T_{ik} = \rho\, u_i u_k + (p - \xi\,\theta)\, h_{ik} - 2\,\eta\,\sigma_{ik} + q_i\, u_k + q_k\, u_i$$
$$+ \frac{1}{4\pi}\left[V_{,i}\, V_k - \frac{1}{2}\, g_{ik}\,(V_{,\ell}\, V^{,\ell} - M^2 V^2) \right] \qquad \ldots(3)$$

2. Formulation of the problem

where

p : the isotropic pressure , ρ: the fluid density

η, ξ: the coefficients of shear and bulk viscosities

u_i : the four vector of the cosmic matter distribution satisfying the relation in co-moving coordinate system,

$$u^i u_i = -1 \quad , \quad u^i = (0,0,0,1) \quad , \quad u_i = (0, H e^x, 0, -1) \quad , \quad h_{ik} = u_i u_k + g_{ik} \qquad \ldots(4)$$

$$\sigma_{ik} = \frac{1}{2}\mu_{ik} - \frac{1}{3}\theta h_{ik}$$: the components of shear tensors and

$$\mu_{ik} = u_{i;k} + u_{k;i} + \dot{u}_i u_k + \dot{u}_k u_i$$

$$\theta = \nabla_u = u^i{}_{;i}$$: the expansion factor

q_i : the heat conduction vector orthogonal u_i , and

$$q_i q^i > 0 \quad , \quad q_i u^i = 0 \qquad \ldots(5)$$

2. Formulation of the problem

However, V is the massive scalar field and we assume that V is a function of *z* only.

We can determine the function V from Klein-Gordon equation for massive scalar field:

$$g_{ik} V_{;ik} + M^2 V = 0 \quad \Rightarrow \quad V(z) = C_1 \sin(M z) + C_2 \cos(Mz) \quad \text{…(6)}$$

C_1 and C_2 are arbitrary constants.

M is related to mass *m* of zero spin particle, $M = \dfrac{2\pi m}{h}$

2. Formulation of the problem

Using co-moving coordinates, the field equations (2) for metric (1) with energy-momentum tensor (3) can be writen as

$$\frac{\ddot{H}}{H}(1-\alpha^2)-\frac{\alpha^2}{4}+\Lambda=-p+(\xi-\frac{2}{3}\eta)\frac{\dot{H}}{H}+\frac{1}{8\pi}(V'^2-M^2V^2) \qquad \text{...(7)}$$

$$\frac{\dot{H}}{H}(\alpha^2-1)=q_1 \quad , \quad q_3 H e^x=0 \qquad \text{...(8)}$$

$$\frac{3}{4}(1-\alpha^2)+\Lambda(\frac{1}{\alpha^2}-1)=-\rho-\frac{p}{\alpha^2}+\frac{1}{\alpha^2}(\frac{4\eta}{3}+\xi)\frac{\dot{H}}{H}+\frac{V'^2}{8\pi}(\frac{1}{\alpha^2}-1)$$
$$+\frac{M^2V^2}{8\pi}(1-\frac{1}{\alpha^2})+\frac{2q_2}{He^x} \qquad \text{...(9)}$$

$$-\frac{3}{4}\alpha^2+1-\Lambda=-\rho-\frac{1}{8\pi}(V'^2-M^2V^2)+\frac{q_2}{He^x} \qquad \text{...(10)}$$

$$\frac{\ddot{H}}{H}(1-\alpha^2)+\frac{\alpha^2}{4}+\Lambda-1=-p+(\xi-\frac{2}{3}\eta)\frac{\dot{H}}{H}-\frac{1}{8\pi}(V'^2-M^2V^2) \qquad \text{...(11)}$$

$$-\frac{3}{4}\alpha^2+1-\Lambda=-\rho-\frac{1}{8\pi}(V'^2-M^2V^2) \qquad \text{...(12)}$$

3. Solution of the problem

Here bulk (ξ) and shear (η) viscosities are positively definite[34]:

$$\eta > 0 \quad , \quad \xi > 0$$

Firstly, from Eq. (12), we get the energy density of cosmic matter distribution:

$$\rho = \frac{3}{4}\alpha^2 + \Lambda - 1 - \frac{V'^2}{8\pi} + \frac{M^2V^2}{8\pi} \qquad \text{...(13)}$$

And from Eqs. (8), (10) and (12)

$$q_1 = (\alpha^2 - 1)\frac{\dot{H}}{H} \quad , \quad q_2 = 0 = q_3 \qquad \text{...(14)}$$

From Eq. (4) and the condition $q_i u^i = 0$, one obtains $q_4 = 0$.

Eqs. (9), (13) and (14) give the pressure of the matter distribution as

$$p = \frac{\alpha^2}{4} - \Lambda + \frac{V'^2}{8\pi} - \frac{M^2V^2}{8\pi} + (\xi + \frac{4}{3}\eta)\frac{\dot{H}}{H} \qquad \text{...(15)}$$

3. Solution of the problem

Using Eqs. (7) and (11), we have

$$\alpha^2 = 2 - \frac{V'^2}{8\pi} \qquad \text{...(16)}$$

From Eqs. (7), (15) and (16) we get the equation

$$(V'^2 - 2\pi)\ddot{H} + 4\pi\eta\dot{H} = 0 \qquad \text{...(17)}$$

and metric potential can be obtained as,

$$H(t) = H_1 + H_2\, e^{-\frac{4\pi\eta}{V'^2 - 2\pi}t} \qquad \text{...(18)}$$

H_1 and H_2 are arbitrary constants. If we take up Eqs. (6), (16) and the condition α is a constant together we have fact that;

$$V = const. \qquad \text{...(19)}$$

3. Solution of the problem

Comparing Eqs. (6) and (19), we have two case;

Case i) $M = 0$ and $\alpha^2 = 2$ (*zero-mass scalar field, V = Const. = C_2).*

Case ii) $M \neq 0$, $z \to 0$ and $\alpha^2 = 2$ (Massive scalar field, *V = Const. = C_2).*

From the energy conservation law ($T^{ik}_{;k} = 0$) for this model, we obtain

$$H_1 = 0 \qquad \text{and} \qquad H(t) = H_2\, e^{2\eta t} \qquad \textbf{...(20)}$$

3. Solution of the problem

Case i) *Solution for zero-mass scalar field ($M = 0$):*

In this case, the matter distribution filled rotating universe is a cosmic matter with zero-mass scalar field.

The study of a such field in general relativity are played very important role to understanding of the nature of scalar meson field like π- and κ- mesons.

Meson as a particle with the charge of elektron and masses of the order of magnitude of 200 electron masses.

3. Solution of the problem

In this solution, fundamental quantities for the model are given by

$$p = \frac{1}{2} - \Lambda + 2\eta(\xi + \frac{4}{3}\eta) \qquad \text{...(21)}$$

$$\rho = \frac{1}{2} + \Lambda \qquad \text{...(22)}$$

$$q_1 = \frac{\dot{H}}{H} = \theta = 2\eta \qquad \text{...(23)}$$

$$\eta = \begin{cases} -\frac{3}{8}\xi + \frac{\sqrt{9\xi^2 - 12 + 24\Lambda}}{8} & \textit{for dust matter } (p = 0) \\ -\frac{3}{8}\xi + \frac{\sqrt{9\xi^2 + 48\Lambda}}{8} & \textit{for stiff matter } (p = \rho) \\ -\frac{3}{8}\xi + \frac{\sqrt{9\xi^2 - 24}}{8} & \textit{for dark matter } (\rho = -p) \\ -\frac{3}{8}\xi + \frac{\sqrt{9\xi^2 - 8 + 32\Lambda}}{8} & \textit{for radiation } (3p = \rho) \end{cases} \qquad \text{...(24)}$$

3. Solution of the problem

For bulk viscsity coefficient (ξ);

$$\xi = \begin{cases} \dfrac{6\Lambda - 3 - 16\eta^2}{12\eta} & for\ \ dust\ \ matter\ \ (p = 0) \\[2ex] \dfrac{3\Lambda - 4\eta^2}{3\eta} & for\ \ stiff\ \ matter\ \ (p = \rho) \\[2ex] -\dfrac{3 + 8\eta^2}{6\eta} & for\ \ dark\ \ matter\ \ (\rho = -p) \\[2ex] \dfrac{4\Lambda - 1 - 8\eta^2}{6\eta} & for\ \ radiation\ \ \ \ (3p = \rho) \end{cases} \quad \text{...(25)}$$

3. Solution of the problem

For the cosmological constant (Λ);

M = 0	Λ	Λ for η = 0	Λ for ξ = 0	Λ for η = 0 = ξ
Dust Matter	$\frac{1}{2}+2\eta(\xi+\frac{4}{3}\eta)$	$\frac{1}{2}$	$\frac{1}{2}+\frac{8}{3}\eta^{2}$	$\frac{1}{2}$
Stiff Matter	$\eta(\xi+\frac{4}{3}\eta)$	0	$\frac{4\eta^{2}}{3}$	0
Dark Matter	-	-	-	-
Radiation	$\frac{1}{4}+\frac{3}{2}\eta(\xi+\frac{4}{3}\eta)$	$\frac{1}{4}$	$\frac{1}{4}+2\eta^{2}$	$\frac{1}{4}$

3. Solution of the problem

Case ii) *Solution for massive-salar field (M ≠ 0):*

In this case, massive scalar field is effective in the beginning of cosmic expansion only, but right after the beginning of expansion the massive scalar field decays rapidly and while matter formation is increased, the pressure of matter distribution is decreased with cosmological constant and massive scalar fields:

$$p = \frac{1}{2} - \Lambda - \frac{M^2 C_2^2}{8\pi} + 2\eta(\xi + \frac{4}{3}\eta) \qquad \textbf{...(26)}$$

$$\rho = \frac{1}{2} + \Lambda + \frac{M^2 C_2^2}{8\pi} \qquad \textbf{...(27)}$$

$$q_1 = \frac{\dot{H}}{H} = \theta = 2\eta \qquad \textbf{...(28)}$$

3. Solution of the problem

For shear and bulk viscosity coefficients:

$$\eta = \begin{cases} -\dfrac{3\xi}{8} - \dfrac{\sqrt{9\pi^2\xi^2 - 12\pi^2(1-2\Lambda) + 3\pi M^2 C_2^2}}{8\pi} & , \text{ for } p = 0 \\ -\dfrac{3\xi}{8} + \dfrac{\sqrt{9\pi^2\xi^2 + 48\pi^2\Lambda + 6\pi M^2 C_2^2}}{8\pi} & , \text{ for } p = \rho \\ -\dfrac{3\xi}{8} + \dfrac{\sqrt{9\xi^2 - 24}}{8} & , \text{ for } \rho = -p \\ -\dfrac{3\xi}{8} + \dfrac{\sqrt{9\pi^2\xi^2 - 8\pi^2(1-4\Lambda) + 4\pi M^2 C_2^2}}{8\pi} & , \text{ for } 3p = \rho \end{cases} \quad \ldots(29)$$

and

$$\xi = \begin{cases} \dfrac{12\pi(2\Lambda - 1) + 3M^2C_2^2 - 64\pi\eta^2}{48\pi\eta} & , \text{ for } p = 0 \\ \dfrac{24\pi\Lambda + 3M^2C_2^2 - 32\pi\eta^2}{24\pi\eta} & , \text{ for } p = \rho \\ -\dfrac{3 + 8\eta^2}{6\eta} & , \text{ for } \rho = -p \\ \dfrac{2\pi(4\Lambda - 1) + M^2C_2^2 - 16\pi\eta^2}{12\pi\eta} & , \text{ for } 3p = \rho \end{cases} \quad \ldots(30)$$

3. Solution of the problem

For the cosmological constant (Λ);

$z \rightarrow 0$	Λ
Dust Matter	$\dfrac{12\pi(1+4\eta\xi)+64\pi\eta^2-3M^2C_2^2}{24\pi}$
Stiff Matter	$\dfrac{8\pi\eta(3\xi+4\eta)-3M^2C_2^2}{24\pi}$
Dark Matter	-
Radiation	$\dfrac{2\pi\left[1+2\eta(3\xi+4\eta)\right]-M^2C_2^2}{8\pi}$

3. Solution of the problem

The phenomenological expression for the heat conduction is given by

$$q_i = \kappa(T_{,k} + T\,\dot{u}_k)h_i^k \quad , \quad h_i^k = \delta_i^k + u_i\,u^k \qquad \textbf{...(31)}$$

where κ is thermal conductivity and T is temperature. From Eqs. (23 or 28), (4) and (31) we get ,

$$q_1 = \kappa\,T_{,1} = \theta = 2\eta \quad , \quad T_{,2} + e^x(T\,\dot{H} - H\,\dot{T}) = 0 \qquad \textbf{...(32)}$$

and

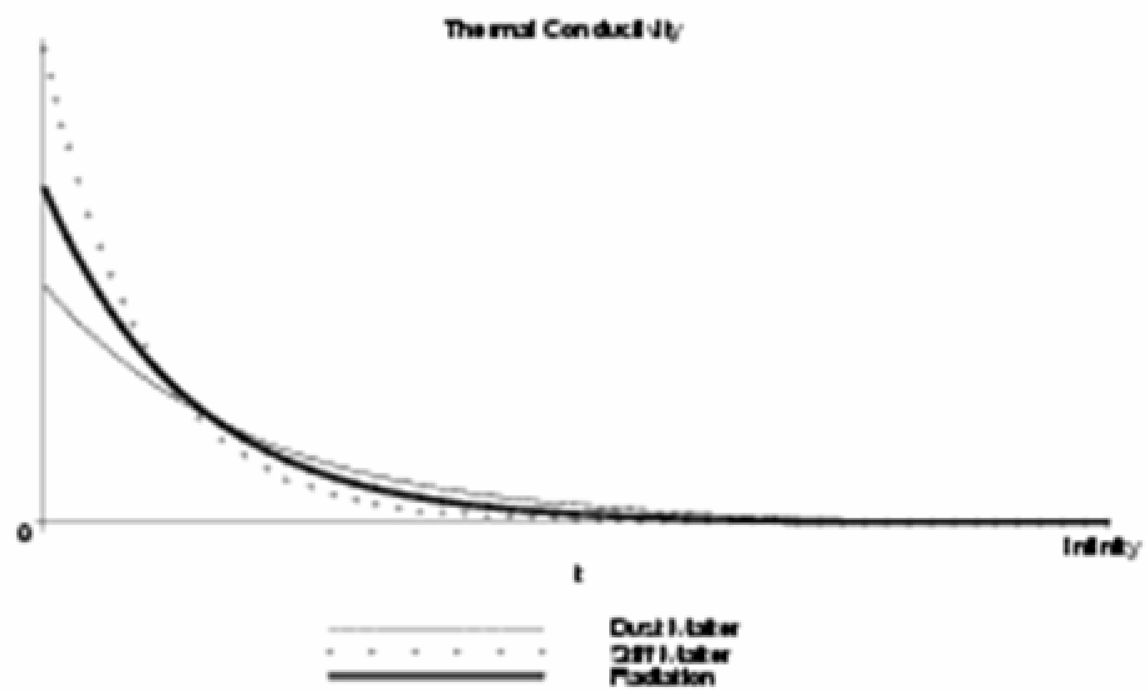
$$\kappa = \frac{\theta}{H\;\beta} = \frac{2\,\eta}{\beta\;H_2 e^{2\eta\,t}} \quad , \quad T = H_2(\beta\,x + \delta)\,e^{2\eta\,t} \qquad \textbf{...(33)}$$

The thermal conductivity and temperature of the cosmic matter distribution in the rotating universe depend on shear viscosity coefficient and cosmic time *t*.

3. Solution of the problem

The relation between the quantities q, κ, T, θ are given as follows:

$$\frac{T}{\theta}=\frac{H_2\, e^{2\eta t}(\beta\, x+\delta)}{2\eta} \quad , \quad \frac{T}{\theta}=\frac{H_2\, e^{2\eta t}(\beta\, x+\delta)}{2\eta} \quad , \quad \frac{\kappa}{\theta}=\frac{1}{\beta\, H_2\, e^{2\eta t}} \qquad \textbf{...(34)}$$



The variation of thermal conductivity for different cosmic matter distributions.

4. Conclusions and Discussion

- In this work, A rotating model universe with massive scalar field, viscous fluid and heat flux is considered in the presence of cosmological term, and for various cosmic matter forms, the behaviour of cosmological term, shear and bulk viscosity coefficients and other kinematical quantities are discussed.

- The solutions have reported that the Λ term plays very important role in rotating universe.

- The proper volume of the universe increases exponentially. This result represents the rotating inflation era in the evoluation of the universe.

$$U^3 = \sqrt{-\mathrm{g}} = \frac{H_2\, e^{2\eta t + x}}{\alpha}$$

4. Conclusions and Discussion

- *In the case i),* from (21)-(23), while the fluid energy density ρ is related to only cosmological term, the pressure is related to cosmological term, shear and bulk viscousity coefficients.
- In this model, cosmological term reduces the pressure while increasing the density of the matter distribution.
- In the period when the dark matter is dominated, it is seen that the cosmological term don’t have an effect on the coefficients of viscosity. Moreover, in this period, the assumption of the coefficients of the viscosity being positive is ignored.

4. Conclusions and Discussion

- In the existence of massive scalar field, it is seen that the decay of massive scalar field, just like the way in the cosmological term, have the effect on decreasing the pressure while increasing the energy density of the cosmic matter distribution. This situation indicates the relationship between the cosmological term and the massive scalar field decay.

4. Conclusions and Discussion

- The thermal conductivity for the matter distribution in the rotating universe decreases rapidly by cosmic expansion. In the radiation era, the thermal conductivity (κ) decays with cosmic time t rapidly than the dust matter era.

- In the presence of massive scalar field, the thermal conductivity decreases rapidly than the zero-mass scalar field case.

4. Conclusions and Discussion

- Finally, scalar fields, as they help in explaining the creation of matter in cosmological theories, represent matter fields with spinless quanta and can describe gravitational fields.

- The investigation of cosmological models with massive scalar field in the presence of cosmological term will be very important to explain the cosmological term problem, dark matter, cosmic expansion, matter creating (*cosmic neutrino, photon and other cosmic forms*), thermodynamical and other cosmological process in inflation era.

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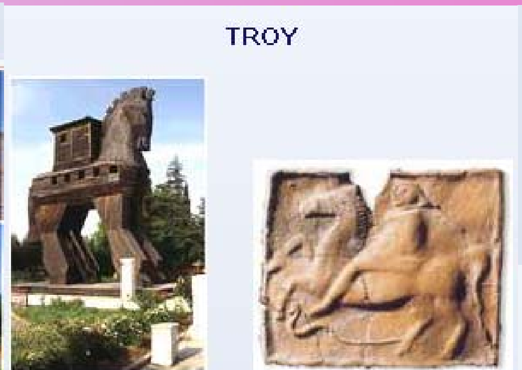

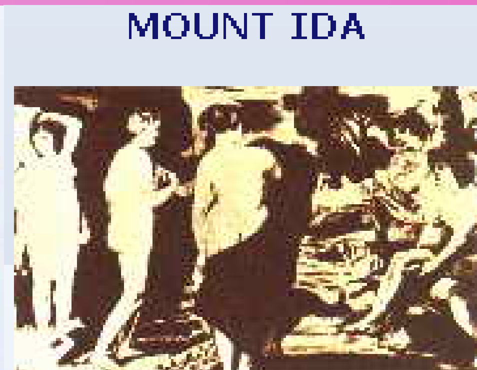

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