12th Regional Conference in Mathematical Physics
Islamabad, Pakistan, March 26 - 1 April 2006

Some New Results in Physics of Dusty Magnetoplasmas

M. Salimullah
Dept. of Physics
Jahangirnagar University
Savar, Dhaka-1342, Bangladesh

1. **Introduction**
2. **Dust-lower-hybrid wave**
3. **EM dust-magnetosonic wave**
4. **Shukla-Nambu-Salimullah potential**
5. **Colloid Crystals in Semiconductors**
6. **Conclusions.**

Dust is ubiquitous

Gas and dust are the main constituents of the Universe.

FOUR SCIENTIFIC COMMUNITIES :

1. **ASTROPHYSICS AND SPACE PHYSICS :**
   – Planetary magnetospheres & rings,
   Comets, Protostars, Molecular clouds,
   Interstellar space, Nebulae, etc.

2. **INDUSTRIAL PLASMA PROCESSES**
   – Wafer contamination in microelectronics,
   Formation and growth of dust in plasma deposition
   and HF etching exptts.
   Agglomeration – fractal structures.

3. **BASIC PLASMA PHYSICS :**
   – WAVES, INSTABILITIES, NEW MODES, DAMPING,
   STRONGLY COUPLED PLASMAS,
   dust-plasma crystals,
   dust-plasma liquids. $\mathbf{KE \ll U}$

4. **EARTH'S ENVIRONMENTS :**
   "NOCTILUCENT CLOUDS" have origin in pollution,
   above 80 km (mesopause)
   "terrestrial arosols" like industrial pollution,
   satellite burning, rocket exhaust ......
   Strong radar backscattering (or, electron "bite-out")
   Global warming (green house effect) may be due to
   dust in the noctilucent cloud.

Our Activities in Plasma Physics

Laser-Plasma Interactions

Microwave radiation in plasmas and semiconductors

Various nonlinear effects

Self-generated mega-gauss magnetic field

Waveguides

Beat Wave Accelerators : Excitations and Instabilities

Realtivistic Effects

Dusty Plasmas : Waves and Instabilities

Dusty Magnetoplasmas

Nonuniform Plasmas

Jeans Instabilities

Wakefields and Dust Crystals

SNS potential

Charging of dust grains

Nonspherical grains

Long-ranged order formation in colloidal plasmas

Dust-lower-hybrid wave in a dusty magnetoplasma

Dust-acoustic waves **(in unmagnetized)** dusty plasmas :

A low-frequency $(\sim 15Hz)$ **and longwavelength** $(\sim 1cm)$ **mode known as dust-acoustic (DA) wave has been extensively studied both theoretically and experimentally :**

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2} - \frac{\omega_{pd}^2}{\omega^2} = 0.$$

Defining

$$C_d = \omega_{pd}\,\lambda_D, \quad \lambda_{Dj}^2 = \frac{T_j}{4\pi n_{0j} e^2}, \quad \omega_{pd}^2 = \frac{4\pi Z_d^2 e^2 n_{do}}{m_d},$$

$$\frac{1}{\lambda_D^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2},$$

the dispersion relation of the dust-acoustic mode is

$$\omega = k\, C_d.$$

However, the magnetic field is invariably present in space plasma systems or can be applied for experimental purposes in laboratory plasmas.

Fluid equations governing the excitation of the plasma modes in general are :

i) Momentum balance equation

$$\frac{\partial \underline{v}_\alpha}{\partial t} + (\underline{v}_\alpha \cdot \underline{\nabla})\underline{v}_\alpha = -\frac{q_\alpha}{m_\alpha}\underline{\nabla}\phi + \underline{v}_\alpha \times \underline{\omega}_{c\alpha} - \frac{v_{t\alpha}^2}{n_{\alpha o}}\underline{\nabla} n_\alpha - \nu_\alpha \underline{v}_\alpha, \tag{1}$$

ii) Equation of continuity

$$\frac{\partial n_\alpha}{\partial t} + \underline{\nabla} \cdot (n_\alpha \underline{v}_\alpha) = 0, \tag{2}$$

iii) Poisson's equation

$$\nabla^2 \phi = -4\pi \sum_\alpha q_\alpha n_\alpha. \tag{3}$$

Solving Eqs.(1-3) by the usual technique, we obtain the linear dielectric function as

$$\epsilon(\omega, \underline{k}) = 1 + \sum_{\alpha=e,i,d} \chi_\alpha, \tag{4}$$

where

$$\begin{aligned} \chi_\alpha \;=\; & \left[\frac{k_\perp^2}{k^2}\frac{\omega_{p\alpha}^2}{\omega_{c\alpha}^2 - \omega'^2}\frac{\omega'}{\omega - k_\parallel u_{o\alpha}} - \frac{k_\parallel^2}{k^2}\frac{\omega_{p\alpha}^2}{\omega'(\omega - k_\parallel u_{o\alpha})}\right] \\ \times\; & \left[1 + \frac{k^2 v_{t\alpha}^2}{\omega_{p\alpha}^2}\left(\frac{k_\perp^2}{k^2}\frac{\omega_{p\alpha}^2}{\omega_{c\alpha}^2 - \omega'^2}\frac{\omega'}{\omega - k_\parallel u_{o\alpha}} - \frac{k_\parallel^2}{k^2}\frac{\omega_{p\alpha}^2}{\omega'(\omega - k_\parallel u_{o\alpha})}\right)\right]^{-1}, \end{aligned} \tag{5}$$

with $\omega' = \omega + i\nu_{\alpha n} - k_\parallel u_{0\alpha}$.

For the low-frequency electrostatic dust-lower-hybrid mode propagating nearly perpendicular to the magnetic field with $\omega_{cd} \ll \omega \ll \omega_{ci} \ll \omega_{ce}$ and $k v_{t\alpha} \ll \omega_{p\alpha}$, we obtain from Eq.(5)

$$\chi_e \;\simeq\; \frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + \frac{i\nu_{en}}{\omega - k_\parallel u_o}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{(\omega - k_\parallel u_o)(\omega - k_\parallel u_o + i\nu_{en})},$$

$$\chi_i \simeq \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{i\nu_{in}}{\omega-k_\parallel u_o}\right)-\frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{(\omega-k_\parallel u_o)(\omega-k_\parallel u_o+i\nu_{in})}, \qquad (6)$$

$$\chi_d \simeq -\frac{k_\perp^2}{k^2}\frac{\omega_{pd}^2}{\omega'^2}\left(1+\frac{i\nu_{dn}}{\omega}\right)-\frac{k_\parallel^2}{k^2}\frac{\omega_{pd}^2}{(\omega-k_\parallel u_o)(\omega-k_\parallel u_o+i\nu_{dn})},$$

where the electrons and ions are assumed to have the same drift velocity, u_o.

For $u_o = 0$ and $\nu_\alpha = 0$, the linear dispersion relation of the DLH mode is obtained for $k_\perp \gg k_\parallel$:

$$1+\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2}-\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}+\frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2}-\frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega^2}-\frac{\omega_{pd}^2}{\omega^2}=0,$$

$$\omega^2=\frac{k^2}{k_\perp^2}\,\frac{\omega_{pd}^2}{\omega_{pi}^2}\,\omega_{ci}^2\,\left(1+\frac{k_\parallel^2}{k^2}\,\frac{\omega_{pd}^2}{\omega_{pe}^2}\right),$$

$$\omega^2 \simeq \omega_{DLH}^2\left[1+\frac{k_\parallel^2}{k^2}\frac{n_{eo}m_d}{Z_d^2 n_{do}m_e}\left(1+\frac{n_{io}m_e}{n_{eo}m_i}\right)\right], \qquad (7)$$

where

$$\omega_{DLH}^2=\frac{\omega_{pd}^2}{\omega_{pi}^2}\,\omega_{ci}^2=\omega_{cd}\omega_{ci}\left(\frac{Z_d n_{do}}{n_{io}}\right)\left(1+\frac{n_{eo}m_e}{n_{io}m_i}\right)^{-1}. \qquad (8)$$

For the collision dominated plasmas, we assume $\nu_{en},\ \nu_{in} > (\omega-k_\parallel u_o)$ and $\omega^2 \gg \nu_{dn}^2$ for the cold dust. Using Eqs.(6), the dispersion relation of the DLH mode is given by

$$\omega^2=\omega_{DLH}^2\left[1+\frac{k_\parallel^2}{k^2}\frac{\omega_{ci}^2}{\nu_{in}^2}\left(1+\frac{n_{eo}T_i}{n_{io}T_e}\right)\right], \qquad (9)$$

where we used $\nu_{in}/\nu_{en} = (T_i m_e/T_e m_i)^{1/2}$.

Using Eqs.(6), the damping rate of this mode for $k_\perp \gg k_\parallel$ is obtained as

$$\frac{\gamma}{\omega}=-\frac{1}{2}\left[\frac{\nu_{dn}}{\omega}+\frac{\nu_{in}}{\omega-k_\parallel u_o}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left\{1+\frac{n_{eo}}{n_{io}}\left(\frac{m_e}{m_i}\frac{T_e}{T_i}\right)^{1/2}\right\}\right]. \qquad (10)$$

Thus, the DLH mode can grow when $u_o > \omega/k_\parallel$ with growth rate determined by Eq.(10). It is noticed from the above equations that the dynamics of electrons is not important for the ion-dust hybrid wave.

Using Vlasov-kinetic equation

$$\epsilon_r \simeq 1 + \frac{1-\Gamma_{oi}}{k^2\lambda_{Di}^2} + \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{\omega_{ci}^2}{\omega_{ce}^2}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2+\omega_{pi}^2}{(\omega-k_\parallel u_o)^2} - \frac{\omega_{pd}^2}{\omega^2}, \quad (11)$$

$$\begin{aligned}\epsilon_i &= \sqrt{\frac{\pi}{2}}\left\{\frac{\omega-k_\parallel u_o}{k_\parallel v_{te}}\frac{1}{k^2\lambda_{De}^2}\exp\left[-\left(\frac{\omega-k_\parallel u_o}{\sqrt{2}k_\parallel v_{te}}\right)^2\right]\right.\\ &+ \left.\frac{\omega-k_\parallel u_o}{k_\parallel v_{ti}}\frac{1}{k^2\lambda_{Di}^2}\exp\left[-\left(\frac{\omega-k_\parallel u_o}{\sqrt{2}k_\parallel v_{ti}}\right)^2\right]\right\}, \quad (12)\end{aligned}$$

where $\Gamma_{oi} = I_o(b_i)\exp(-b_i)$, $b_i = k_\perp^2 v_{ti}^2/\omega_{ci}^2$, and I_o is the zero-order modified Bessel function of argument (b_i).

Damping rate of the mode is given by

$$\gamma_L = -\omega_{DLH}^2\epsilon_i/2\omega_{pd}^2. \quad (13)$$

For the usual parameters in laboratory experiments, $m_d/m_i \sim 10^{12}$, $B_s \sim 1kG$ one can obtain $\omega_{ci} \sim 10^6\ Hz$ and $\omega_{cd} \sim 10^{-2}\ Hz$. Thus, the dust cyclotron frequency will be too small to be detected in the laboratory conditions. It may be significant in space environments. However, the dust-lower-hybrid frequency may take a significant value, $\omega_{DLH} \sim 10^2\ Hz$ for $Z_d n_{do}/n_{io} = 1$.

Dust-lower-hybrid instability

The dielectric response function ($\omega \ll \omega_{ci}$)

$$\epsilon(\omega, \underline{k}) = 1 + \frac{1}{k^2 \lambda_{De}^2} + \frac{k_\perp^2}{k^2}\ \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{k_\parallel^2}{k^2} \frac{\omega_{pi}^2}{(\omega - k_\parallel u_{i0})^2} - \frac{\omega_{pd}^2}{(\omega - k_\parallel v_o)^2}. \tag{14}$$

The dusty plasmas support the following two dust-modes under the specific condition :

i) Dust-acoustic wave, $\left(1 + \frac{1}{k^2 \lambda_{De}^2}\right) \gg \frac{k_\perp^2}{k^2} \frac{\omega_{pi}^2}{\omega_{ci}^2}$,

$$\omega^2 = \frac{k^2 C_d^2}{(1 + k^2 \lambda_{De}^2)} \left(1 + \frac{k_\parallel^2}{k^2} \frac{\omega_{pi}^2}{\omega_{pd}^2}\right), \tag{15}$$

where $C_d = \omega_{pd} \lambda_{De}$. Here we assume $k_\parallel \gg k_\perp$ for almost parallel propagation of the dusty plasma mode. Consequently, the modification of the mode by the external magnetic field will be negligible.

ii) Dust-lower-hybrid wave, $\left(1 + \frac{1}{k^2 \lambda_{De}^2}\right) \ll \frac{k_\perp^2}{k^2} \frac{\omega_{pi}^2}{\omega_{ci}^2}$,

$$\omega^2 = \frac{k^2}{k_\perp^2} \omega_{dlh}^2 \left(1 + \frac{k_\parallel^2}{k^2} \frac{\omega_{pi}^2}{\omega_{pd}^2}\right), \tag{16}$$

where $\omega_{dlh} = \omega_{ci} \omega_{pd} / \omega_{pi}$.

These conditions are usually valid for relatively high density plasma ($\omega_{pi} \gg \omega_{ci}$) and for almost perpendicular propagation ($k_\perp \sim k$). This case represents an ion-dust plasma where the electrons are "eaten up" by the dust grains on sticking collisions. Here, the magnetic field on ions plays larger role than that of the thermal effect of the electrons.

Phys. Scr. 73, 169 (2006).

Phys. Plasmas 9, 5121 (2002).

Magnetosonic Wave Instability In a Streaming Dusty Plasma

Waves and instabilities occupy the major part of basic research in dusty plasma physics in recent years.

EM Magneto-acoustic waves in a Magnetized Dusty Plasma

Vlasov equation:

$$\frac{\partial f_\alpha}{\partial t} + \underline{v} \cdot \underline{\nabla} f_\alpha + \frac{q_\alpha}{m_\alpha} \left(\underline{E} + \frac{\underline{v} \times \underline{B}^T}{c} \right) \cdot \frac{\partial f_\alpha}{\partial \underline{v}} = 0. \tag{17}$$

By definition

$$\begin{aligned} \underline{J}_{\underline{k}\omega} &= \sum_\alpha n_{\alpha o} q_\alpha \int \underline{v} f_{\alpha \underline{k}\omega} d\underline{v}, \\ &= \sum_\alpha \left(- \frac{i n_{\alpha o} q_\alpha^2}{2T_\alpha} \right) \sum_{n,l} \frac{\underline{v} J_n(\rho_\perp) \exp\left[i(n-l)\phi\right]}{(l\omega_{c\alpha} + k_\parallel v_\parallel - \omega)} \; A_{l\underline{k}\omega} f_{\alpha o} d\underline{v}, \\ &\equiv \underline{\underline{\sigma}} \cdot \underline{E}. \end{aligned} \tag{18}$$

The components of the conductivity tensor can be written down. Generally,we can write

$$\sigma_{\mu\nu} = \sum_\alpha \left(\frac{-i n_\alpha q_\alpha^2}{2T_\alpha} \right) \sum_{n,l} \int \frac{v_\mu a_\nu J_n(\rho_\perp) \exp\left[i(n-l)\phi\right]}{(l\omega_{c\alpha} + k_\parallel v_\parallel - \omega)} \; f_{\alpha o} \, d\underline{v}, \tag{19}$$

with $\mu, \nu = x, \, y \, z$. **One can immediately write down the components of the dielectric tensor from**

$$\underline{\underline{\epsilon}} = \underline{\underline{I}} + \frac{4\pi i}{\omega} \underline{\underline{\sigma}}. \tag{20}$$

Using the Maxwell's curl equations and the definition of the current density, we can write down the wave equation in the form

$$-\underline{k} \times (\underline{k} \times \underline{E}_{\underline{k}\omega}) = \frac{\omega^2}{c^2} \underline{E}_{\underline{k}\omega} + \frac{4\pi i \omega}{c^2} \underline{J}_{\underline{k}\omega}, \tag{21}$$

or,

$$\underline{\underline{D}} \cdot \underline{E} = 0. \tag{22}$$

where

$$\underline{\underline{D}} = k^2 \underline{\underline{I}} - \underline{kk} - \frac{\omega^2}{c^2} \underline{\underline{\epsilon}}, \tag{23}$$

$\underline{\underline{I}}$ **being the unit dyadic.**

The general dispersion relation of any wave either electrostatic or electromagnetic, in a flowing plasma is given by

$$\mid \underline{\underline{D}} \mid = 0. \tag{24}$$

We finally obtain the plasma dispersion tensor in the magnetized dusty plasma as

$$\underline{\underline{D}} \equiv \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}, \tag{25}$$

where the components of the plasma dispersion tensor $\underline{\underline{D}}$ are

$$\begin{aligned} D_{xx} &= 1 - \frac{k_\parallel^2 c^2}{\omega^2} + \frac{4\pi i}{\omega}\sigma_{xx}, \\ D_{xy} &= \frac{4\pi i}{\omega}\sigma_{xy}, \\ D_{xz} &= \frac{k_\perp k_\parallel c^2}{\omega^2} + \frac{4\pi i}{\omega}\sigma_{xz}, \\ D_{yx} &= \frac{4\pi i}{\omega}\sigma_{yx}, \\ D_{yy} &= 1 - \frac{k^2 c^2}{\omega^2} + \frac{4\pi i}{\omega}\sigma_{yy}, \\ D_{yz} &= \frac{4\pi i}{\omega}\sigma_{yz}, \\ D_{zx} &= \frac{k_\perp k_\parallel c^2}{\omega^2} + \frac{4\pi i}{\omega}\sigma_{zx}, \\ D_{zy} &= \frac{4\pi i}{\omega}\sigma_{zy}, \\ D_{zz} &= 1 - \frac{k_\perp^2 c^2}{\omega^2} + \frac{4\pi i}{\omega}\sigma_{zz}. \end{aligned} \tag{26}$$

Equation (24) can be simplified to study the properties of a particular mode in a flowing magnetized multicomponent plasma.

Magnetosonic Wave Instability

Let us consider the propagation of an EM wave nearly perpendicular to the external static magnetic field having a small but finite $k_\parallel$ in a flowing dusty plasma. We assume $\omega \ll \omega_{ci}$, $E_z = E_x = 0$, $E_y \neq 0$, $\underline{k} \simeq \hat{x}k_\perp + \hat{z}k_\parallel$, $k_\perp^2 \gg k_\parallel^2$. From Eq.(24), the dispersion relation of the magnetosonic wave is given by

$$\frac{k^2c^2}{\omega^2} = 1 + \chi_{yy}, \tag{27}$$

where

$$\begin{aligned}\chi_{yy} &= \frac{4\pi i}{\omega}\,\sigma_{yy}, \\ &= \sum_\alpha \sum_l \frac{4\pi\omega_{p\alpha}^2}{\omega v_{t\alpha}^2}\frac{1}{\pi\sqrt{\pi}v_{t\alpha}^3} I_\perp\, I_\parallel, \end{aligned} \tag{28}$$

with

$$I_\perp = \int_o^\infty v_\perp^3 J_l'^2\left(\frac{k_\perp v_\perp}{\omega_{c\alpha}}\right)\, \exp\left[-v_\perp^2/v_{t\alpha}^2\right] dv_\perp, \tag{29}$$

$$I_\parallel = \int_{-\infty}^\infty \frac{\exp\left[-(v_\parallel - u_{\alpha o})^2/v_{t\alpha}^2\right]}{(l\omega_{c\alpha} + k_\parallel v_\parallel - \omega)}\; dv_\parallel. \tag{30}$$

Thus,

$$\frac{k^2c^2}{\omega^2} = 1 + \sum_\alpha\sum_l \frac{\omega_{p\alpha}^2}{\omega\, k_\parallel\, v_{t\alpha}}\frac{\exp\left(-b_\alpha\right)}{b_\alpha}\;\, l^2\, I_l(b_\alpha)\, Z(\xi_{l\alpha}), \tag{31}$$

where I_l is the modified Bessel function of order l, $b_\alpha = k_\perp^2 v_{t\alpha}^2/2\omega_{c\alpha}^2$, and $\xi_{l\alpha} = (\omega - k_\parallel u_{\alpha o} - l\omega_{c\alpha})/k_\parallel v_{t\alpha}$.

When dust is taken unmagnetized and electrons and ions are assumed magnetized $(\omega_{cd} \ll \omega \ll \omega_{ci} \ll \omega_{ce})$, we obtain

$$\frac{k_\perp^2 c^2}{\omega_r^2} = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pd}^2}{\omega^2}. \tag{32}$$

Thus, for high density plasma limit, $\omega_{pi}^2/\omega_{ci}^2 \gg \omega_{pe}^2/\omega_{ce}^2 \gg 1$, we obtain the dispersion relation

$$\omega^2 = \omega_{dlh}^2 + k_\perp^2 v_A^2, \tag{33}$$

where $\omega_A = c\omega_{ci}/\omega_{pi}$ and the dust-lower-hybrid frequency and is given by $\omega_{dlh}^2 = \omega_{pd}^2\omega_{ci}^2/\omega_{pi}^2$.

This is the usual EM magnetosonic wave, modified by the presence of the unmagnetized dust grains, having a cutoff at ω_{dlh}. Obviously, this low-frequency EM mode reduces to the usual magnetosonic wave in an electron-ion plasma $(\omega^2 = k_\perp^2 v_\perp^2)$ in absence of the dust.

M. Salimullah and G.E. Morfill, Phys. Rev. E 59, R 2558 (1999);

P. K. Shukla, M. Salimullah, and G. Sorasio, Phys. Plasmas 9, 5121 (2002).

M. Salimullah and M. Rosenberg, Phys. Lett. A 254, 347 (1999).

On the Shukla-Nambu-Salimullah potential in magnetized plasmas

In magnetized plasmas, there are two electrostatic potentials, viz., the almost spherically symmetric Debye-Hückel and the strongly anisotropic Shukla-Nambu-Salimullah (SNS) potentials. The physics of the later potential is due to the ion polarization drift. The exact solution for the SNS-potential introduces a new shielding length across the external magnetic field which is much larger than that of the Debye-Hückel potential.

The existence of this new electrostatic potential in magnetized plasmas was first pointed out in several papers :

M. Salimullah and P.K. Shukla, Phys. Plasmas 5, 4205 (1998).

M. Salimullah and M. Nambu,
J. Phys. Soc. Japan 69, 1688 (200);
Phys. Lett. A286, 418 (2001);
Phys. Rev. E 63, o56 403 (2001).

P.K. Shukla, M. Nambu, and M. Salimullah,
Phys. Lett. A 291, 413 (2001).

M. Salimullah, P.K. Shukla, M. Nambu, O. Ishihara, and A.M. Rizwan,
Phys. Plasmas 10, 3047 (2003).

H. Nitta, M. Nambu, M. Salimullah, and P.K. Shukla,
Phys. Lett. A 308, 451 (2003).

The S-N-S potential originates from the modified ion-acoustic waves which are the electrostatic lf $(\omega \ll \omega_{ci})$ waves. Since the modified ion-acoustic waves are the mixing mode between the ion-acoustic and ion-cyclotron waves, which propagate obliquely to the external magnetic field direction, they couple with the ions polarization drifts. In a magnetized plasma, the ions perform the polarization drifts with drift velocity $\underline{v}_d = i(d\underline{E}_\perp/dt)/B_o\omega_{ci}$.

The dispersion relation of the modified ion-acoustic waves are $\omega = \pm\omega_k = \pm k_\parallel C_s/(1+k_\perp^2\rho_s^2)^{1/2}$, where $C_s = \omega_{pi}\lambda_{De}, \ \ \rho_s = C_s/\omega_{ci}$.

Dielectric Response Function

For the Maxwellian electron-ion plasma $(T_e \gg T_i)$ with electrons as the Boltzmann gas, ions are governed by

$$\begin{aligned} \frac{\partial \underline{v}_i}{\partial t} + (\underline{v}_i \cdot \underline{\nabla})\underline{v}_i &= \frac{e\underline{E}}{m_i} + \underline{v}_i \times \underline{\omega}_{ci} - \underline{\nabla} p_i, & (34) \\ \frac{\partial n_i}{\partial t} + \underline{\nabla} \cdot n_i \underline{v}_i &= 0, \ \ p_i = n_i k_B T_i. & (35) \end{aligned}$$

Thus,

$$\begin{aligned} \epsilon(\omega, \underline{k}) = 1 + \frac{1}{k^2\lambda_{De}^2} &+ \frac{k_\perp^2}{k^2} \cdot \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2 + k^2 v_{ti}^2 - k_\parallel^2 v_{ti}^2 \omega_{ci}^2/\omega^2} \\ &+ \frac{k_\parallel^2}{k^2} \cdot \frac{\omega_{pi}^2(1-\omega_{ci}^2/\omega^2)}{\omega_{ci}^2 - \omega^2 + k^2 v_{ti}^2 - k_\parallel^2 v_{ti}^2 \omega_{ci}^2/\omega^2}. \qquad (36) \end{aligned}$$

For $\omega \ll \omega_{ci}$ and $k_\parallel = 0,\ k_\perp = k$, one can obtain

$$\omega(k) = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2 + k^2 v_{ti}^2}. \tag{37}$$

The condition $\epsilon(k) = 0$ yields a fourth-order equation in k

$$k^4 + Ak^2 + B = 0, \tag{38}$$

where

$$A = (\omega_{ci}^2 + k_e^2 v_{ti}^2 + \omega_{pi}^2)/v_{ti}^2,\ \ B = k_e^2\omega_{ci}^2/v_{ti}^2. \tag{39}$$

For $f = \omega_{pi}^2/\omega_{ci}^2 \gg 1$ and $C_s \gg v_{ti}$, $B/A^2 \ll 1$ and

$$k_\pm^2 = \frac{-A \pm A(1 - 2B/A^2)}{2}, \quad k_e = 1/\lambda_{De}. \tag{40}$$

The + branch is

$$k_+ = \pm\frac{ik_e}{\sqrt{f(1 + v_{ti}^2/C_s^2) + 1}}, \tag{41}$$

which corresponds to the S-N-S potential modified by the ion thermal velocity.

The other solution gives the usual Debye-Hückel potential modified by the external magnetic field

$$k_- = \pm i\sqrt{k_e^2 + k_i^2(1 + 1/f)}. \tag{42}$$

It should be mentioned here that the S-N-S potential originates from the ion polarization drift in magnetized plasmas in contrast to the Debye-Hückel potential which comes from the quasi-neutrality. For $f \gg 1$, the ion-polarization drift should dominate in comparison with the contribution coming from the departure from the quasi-neutrality.

EXACT SNS POTENTIAL :
For the modified ion-acoustic waves

$$\epsilon(\omega, \underline{k}) = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{k_\perp^2}{k^2}\;\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega^2}. \tag{43}$$

$$\begin{aligned}\Phi(\underline{x}, t) &= \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \underline{k}\cdot\underline{v}_t)}{\epsilon(\omega, \underline{k})}\exp\left(i\underline{k}\cdot\underline{r}\right)\; d\mathbf{k}d\omega, \\ &= \int \frac{q_t}{\pi}\;\frac{\delta(\omega - k_\parallel v_t)\; J_0(k_\perp\rho)e^{ik_\parallel(z-v_t t)}\;\; k_\perp\, dk_\perp\, d\omega}{k^2\epsilon(\omega, \mathbf{k})}, \\ &= \frac{q_t}{\pi}\;\int \frac{J_0(k_\perp\rho)e^{ik_\parallel(z-v_t t)}\; k_\perp\, dk_\perp\, dk_\parallel}{k^2\epsilon(\omega, \mathbf{k})},\end{aligned} \tag{44}$$

where $f \equiv \omega_{pi}^2/\omega_{ci}^2$.

FINALLY,

$$\Phi(\mathbf{x}, t) = \frac{q_t}{\sqrt{f+1}}\frac{\exp\left[-\sqrt{(1-M^{-2})(f+1)}\;\sqrt{(f+1)|z-v_t t|^2+\rho^2}/\lambda_{De}\right]}{\sqrt{(f+1)|z-v_t t|^2+\rho^2}}. \tag{12}$$

FOR $\rho = 0$**,**

$$\Phi(\xi) = \frac{q_t}{\sqrt{1+f}}\frac{\exp\left(-\xi/L_\parallel\right)}{\xi}, \quad L_\parallel = \frac{\lambda_{De}}{\sqrt{1-M^{-2}}}. \tag{13}$$

FOR $\xi = 0$**,**

$$\Phi(\rho) = \frac{q_t}{\sqrt{1+f}}\frac{\exp\left(-\rho/L_\perp\right)}{\rho}, \quad L_\perp = \sqrt{\frac{1+f}{1-M^{-2}}}\quad \lambda_{De}. \tag{14}$$

THUS, SNS POTENTIAL IS ELLIPTICAL IN SHAPE ELONGATED ACROSS THE EXTERNAL MAGNETIC FIELD.

Phys. Lett. A 300, 82 (2002).

On Shukla-Nambu-Salimullah potential in a streaming dusty magnetoplasma

The appropriate dielectric constant of electrostatic waves in a magnetoplasma is

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{k_\perp^2}{k^2}\,\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{(\omega - k_\parallel u_{i0})^2} - \frac{\omega_{pd}^2}{(\omega - k_\parallel V_0)^2}. \tag{1}$$

$$\omega^2 = \frac{k^2 C_d^2 + k_\parallel^2 C_s^2}{1 + k^2\lambda_{De}^2 + k_\perp^2 f \lambda_{De}^2}, \tag{1a}$$

where $C_d = \omega_{pd}\lambda_{De},\ C_s = \omega_{pi}\lambda_{De},$ **and** $f = \omega_{pi}^2/\omega_{ci}^2$**.**

The electrostatic potential around a test dust particulate is

$$\Phi(\mathbf{x}, t) = \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega, \mathbf{k})}\ \exp(i\mathbf{k}\cdot\mathbf{r})\, d\mathbf{k}\, d\omega, \tag{2}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t,\ \mathbf{v}_t$ **is the velocity vector of a test dust particulate, and** q_t **is its charge.**

Substituting Eq. (1) into Eq. (2) and performing the θ**-integration, we obtain the total electrostatic potential in the cylindrical coordinates** (ρ, θ, z)

$$\Phi(\rho, \xi) = \frac{q_t}{\pi}\int \frac{\delta(\omega - k_\parallel v_t) J_0(k_\perp \rho)\exp(ik_\parallel \xi)\, k_\perp\, dk_\perp\, d\omega\, dk_\parallel}{k^2 + k_e^2 + k_\perp^2 f - k_\parallel^2\omega_{pi}^2/(\omega - k_\parallel u_{i0})^2 - k^2\omega_{pd}^2/(\omega - k_\parallel V_0)^2}, \tag{3}$$

where $k_e = 1/\lambda_{De},\ \xi \equiv z - v_t t$**.**

On performing the ω**-integration, one readily obtains**

$$\Phi(\rho, \xi) = \frac{q_t}{\pi}\int \frac{k_\parallel^2\ J_0(k_\perp\rho)\ \exp(ik_\parallel\xi)\, k_\perp\, dk_\perp\, dk_\parallel}{k_\parallel^4 + k_\parallel^2\{k_\perp^2(f+1) + k_e^2(1 - M_1^{-2} - M_2^{-2})\} - k_e^2 k_\perp^2 M_2^{-2}}, \tag{4}$$

where $M_1 = |v_t - u_{i0}|/C_s$ **and** $M_2 = |v_t - V_0|/C_d$**.**

It is noted that the denominator of Eq. (4) is quadratic in $k_\parallel^2$**. Introducing the dimensionless notation** $\mathbf{K} = \mathbf{k}\lambda_{De}$**, Eq. (4) can be**

rewritten as

$$\Phi(\rho,\xi) = \frac{q_t}{\pi\lambda_{De}} \int \frac{K_\parallel^2 J_0(K_\perp \rho/\lambda_{De}) \exp\left(iK_\parallel \xi/\lambda_{De}\right) K_\perp \, dK_\perp \, dK_\parallel}{(K_\parallel^2 + K_0^2)(K_\parallel^2 - K_1^2)}, \tag{5}$$

where

$$K_{0,1}^2 = \pm\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 + b},$$
$$a = K_\perp^2(f+1) + 1 - M_1^{-2} - M_2^{-2}, \tag{6}$$
$$b = K_\perp^2 M_2^{-2}.$$

Breaking into partial fractions, Eq. (5) yields

$$\Phi_I(\mathbf{x},t) = \left(-\frac{q_t}{\pi\lambda_{De}}\right) \int \frac{K_\parallel^2 \, J_0(K_\perp \rho/\lambda_{De}) \exp\left(iK_\parallel \xi/\lambda_{De}\right) K_\perp \, dK_\perp \, dK_\parallel}{(K_0^2 + K_1^2)(K_\parallel^2 + K_0^2)}, \tag{7}$$

$$\Phi_{II}(\mathbf{x},t) = \left(\frac{q_t}{\pi\lambda_{De}}\right) \int \frac{K_\parallel^2 \, J_0(K_\perp \rho/\lambda_{De}) \exp\left(iK_\parallel \xi/\lambda_{De}\right) K_\perp \, dK_\perp \, dK_\parallel}{(K_0^2 + K_1^2)(K_\parallel^2 - K_1^2)}, \tag{8}$$

Evaluating $K_\parallel$-integration, the new Debye-shielding and dynamical potentials can be obtained from Eqs. (7) and (8) as

$$\Phi_I(\rho,\xi) = \left(\frac{q_t}{\lambda_{De}}\right) \int_0^{K_c} \frac{K_0}{K_0^2 + K_1^2} J_0\left(\frac{K_\perp \rho}{\lambda_{De}}\right) \exp\left(-K_0 \xi/\lambda_{De}\right) K_\perp \, dK_\perp, \tag{9}$$

$$\Phi_{II}(\rho,\xi) = \left(-\frac{2q_t}{\lambda_{De}}\right) \int_0^{K_c} \frac{K_1}{K_0^2 + K_1^2} J_0\left(\frac{K_\perp \rho}{\lambda_{De}}\right) \, Sin\left(\frac{K_1 \xi}{\lambda_{De}}\right) \, K_\perp \, dK_\perp. \tag{10}$$

Our results show that both repulsive SNS screening and attractive dynamical wake potentials are drastically affected by the magnetic field.

In conclusion, we stress that the knowledge of the newly found interaction potentials are a necessary prerequisite for designing new laboratory experiments, so that robust dust-Coulomb crystals in the presence of an external magnetic fiield could be fabricated.

References

1. **M. Nambu and H. Nitta, Phys. Lett. A 300, 82 (2002).**

2. **P. K. Shukla, M. Nambu, and M. Salimullah, Phys. Lett. 291, 413 (2001).**

3. **M. Salimullah and M. Nambu, J. Phys. Soc. Japan 69, 1688 (2000).**

4. **M. Salimullah and P. K. Shukla, Phys. Plasmas 5 , 4205 (1998).**

5. **R. Bharuthram and M. Rosenberg, Planet. Space Sci. 46, 425 (1998).**

6. **M. Salimullah and P.K. Shukla, Phys. Plasmas 6, 686 (1999).**

7. **A.B. Milkhailovskii and V.A. Klimenko, J. Plasma Phys. 24, 385 (1980).**

8. **X.Y. Wang, Z.X. Lin, Z.Y. Li, and X.B. Zhang, Phys. Plasmas 5, 836 (1998).**

9. **M. Salimullah, P. K. Shukla, M. Nambu, H. Nitta, O. Ishihara, and A. M. Rizwan, Phys. Plasmas 10, 3047 (2003).**

10. **D. S. Lemons, M. S. Murillo, W. Daughton, and D. Winske, Phys. Plasmas 7, 2306 (2000).**

11. **O. Ishihara and S. V. Vladimirov, Phys. Plasmas 4, 69 (1997).**

12. **N. Sato, G. Uchida, and T. Kaneko** *et al.*, **Phys. Plasmas 8, 1786 (2001).**

Long-ranged Order Formation of Colloids of Implanted Ions in a DC Biased Piezoelectric Semiconductor

A dc bias in a piezoelectric semiconductor may drive a beam of electrons which could charge the neutralized colloids of implanted ions and cause a uniform drift of charged colloidal particles.

The periodic wakefield may cause a long-ranged ordered structure of charged colloidal particles within the semiconductor to exhibit various additional properties.

In the presence of the uniform dc bias E_d, the ratio of V_0 to u_{e0} is

$$\frac{V_0}{u_{e0}} = \frac{qE_d/m_d\nu_{dn}}{eE_d/m_e\nu_{en}} = \frac{q}{e}\frac{m_e}{m_d}\frac{\nu_{en}}{\nu_{dn}}. \tag{1}$$

WAKE POTENTIALS

Using the standard plasma fluid equations for the Doppler-shifted electrons and colloid particles and including the electron-phonon coupling effect, the dielectric constant of the piezoelectric semiconductor plasma is given by

$$\epsilon(\omega, \mathbf{k}) = \epsilon_L + \frac{i\omega_{pe}^2}{\omega' \left(\nu_0 - i\omega' + ik^2 v_{te}^2/\omega'\right)} - \frac{S^2 k^2 C_s^2}{\omega^2 - k^2 C_s^2} - \frac{\omega_{pd}^2}{(\omega - \mathbf{k}\cdot\mathbf{V}_0)^2}, \tag{2}$$

where $\omega' = \omega - \mathbf{k}\cdot\mathbf{u}_{e0}$. The third term in the right-hand side of Eq. (2) is the piezoelectric contribution from the lattice where S is the dimensionless electromechanical coupling coefficient. The numerical value of S^2 for most of the piezoelectric semiconductors is $\approx 10^{-3}$.

The electrostatic potential around an isolated test charged particulate is given by

$$\phi(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{k^2\epsilon(\mathbf{k},\omega)}\ \exp\left[i\mathbf{k}\cdot\mathbf{r}\right] d\mathbf{k}\, d\omega, \tag{3}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of a test charged particulate, and q_t is its charge.

We may then rewrite Eq. (2) as

$$\epsilon(\omega,\mathbf{k}) = \frac{1+\epsilon_L k^2\lambda_{De}^2}{k^2\lambda_{De}^2}\left[1 - \frac{\{\omega_{k1}^2(\omega - k_\parallel V_0)^2 + \omega_{k2}^2(\omega^2 - k^2C_s^2)\}}{(\omega^2 - k^2C_s^2)(\omega - k_\parallel V_0)^2}\right], \tag{4}$$

where

$$\omega_{k1}^2 = \frac{S^2k^4C_s^2\lambda_{De}^2}{1+\epsilon_L k^2\lambda_{De}^2}, \tag{5}$$

$$\omega_{k2}^2 = \frac{k^2C_d^2}{1+\epsilon_L k^2\lambda_{De}^2}, \tag{6}$$

and $C_d \equiv \omega_{pd}\lambda_{De}$.

The inverse of the real part of the dielectric constant, $\epsilon(\omega,\mathbf{k})$, can be written as

$$\frac{1}{\epsilon(\omega,\mathbf{k})} = \frac{k^2\lambda_{De}^2}{1+\epsilon_L k^2\lambda_{De}^2}\left[1 + \frac{\{\omega_{k1}^2(\omega - k_\parallel V_0)^2 + \omega_{k2}^2(\omega^2 - k^2C_s^2)\}}{(\omega^2 - k^2C_s^2)(\omega - k_\parallel V_0)^2 - \{\omega_{k1}^2(\omega - k_\parallel V_0)^2 + \omega_{k2}^2(\omega^2 - k^2C_s^2)\}}\right]. \tag{7}$$

Substituting Eq. (7) into Eq. (3) and following the standard mathematical techniques, we obtain the total electrostatic potential

$$\Phi = \Phi_I + \Phi_{II} + \Phi_{III}, \tag{8}$$

where

$$\phi_I = \left(\frac{q_t}{r\epsilon_L}\right)\ \exp\left(-\frac{r}{\sqrt{\epsilon_L}\lambda_{De}}\right) \tag{9}$$

is the static Debye-Hückel screening potential with the effective screening length $\sqrt{\epsilon_L}\lambda_{De}$ and the effective charge q_t/ϵ_L. We now use (ρ,θ,z) as the cylindrical coordinates of $\mathbf{r}$, where $r = \left(\rho^2+z^2\right)^{1/2}$.

$$\phi_{II}(\rho,z,t) = \left(\frac{q_t^2S^2\lambda_{De}^4C_s^2}{2\pi^2}\right)\times$$

$$\int \frac{k^4 \,(\omega - k_\parallel V_0)^2 \,\delta(\omega - k_\parallel v_t) \exp{(i\mathbf{k}\cdot\mathbf{r})} \;\, d^3k \, d\omega}{D}, \tag{10}$$

$$D = (1+\epsilon_L k^2\lambda_{De}^2)[(\omega^2 - k^2C_s^2)(\omega - k_\parallel V_0)^2(1+\epsilon_L k^2\lambda_{De}^2) - (\omega - k_\parallel V_0)^2 S^2 k^4 C_s^2 \lambda_{De}^2 - (\omega^2$$

where $\xi = z - v_t t$ and $\mathbf{v}_t \parallel \hat{z}$ is assumed.

Performing $\omega-$ and $\theta-$integrations in Eq. (10), we readily obtain

$$\Phi_{II}(\rho, z, t) = \left(\frac{q_t^2 S^2 C_s^2 \lambda_{De}^4}{\pi}\right)$$

$$\times \int \frac{J_0(k_\perp \rho) \;\, k^4 \, k_\parallel^2 \,\exp{(ik_\parallel \xi)} \, k_\perp dk_\perp \, dk_\parallel}{(1+\epsilon_L k^2\lambda_{De}^2)(k_\parallel^2 v_t^2 - k^2 C_s^2)\{k_\parallel^2(1+\epsilon_L k^2\lambda_{De}^2) - k^2 C_d^2(v_t - V_0)^{-2}\}}. \tag{11}$$

Introducing the dimensionless notation $\mathbf{K} = \mathbf{k}\sqrt{\epsilon_L}\;\lambda_{De}$, we can write

$$\Phi_{II}(\rho, \xi) = \left(\frac{q_t^2 S^2 C_s^2}{\pi \epsilon_L^2 \sqrt{\epsilon_L}\lambda_{De}}\right) \int_0^\infty dK_\perp K_\perp J_0(K_\perp \rho/\sqrt{\epsilon}\lambda_{De}) \; I_\parallel, \tag{12}$$

where

$$I_\parallel = \int_{-\infty}^{\infty} \frac{(K_\parallel^2 + K_\perp^2)^2 K_\parallel^2 \exp{(iK_\parallel \xi/\sqrt{\epsilon}\lambda_{De})}\, dK_\parallel}{(1+K_\parallel^2+K_\perp^2)(K_\parallel^2 v_t^2 - K^2C_s^2)\{K_\parallel^2(1+K^2) - K^2C_d^2(v_t - V_0)^{-2}\}}. \tag{13}$$

Since the last factor of the denominator of Eq. (13) is quadratic in $K_\parallel^2$, we can write

$$K_\parallel^2(1+K_\parallel^2+K_\perp^2) - (K_\parallel^2+K_\perp^2)C_d^2(v_t - V_0)^{-2} \equiv (K_\parallel^2 + K_0^2)(K_\parallel^2 - K_1^2), \tag{14}$$

where

$$K_{0,1}^2 = \pm\frac{1+K_\perp^2 - M^{-2}}{2} + \sqrt{K_\perp^2 M^{-2} + (1+K_\perp^2 - M^{-2})^2/4}, \tag{15}$$

and $M \equiv (v_t - V_0)/C_d$. For $K_\perp < 1$ and $M > 1$

$$K_0^2 \approx 1, \;\; K_1^2 \approx K_\perp^2 M^{-2}. \tag{16}$$

Carrying out the $K_\parallel$-integration, we have

$$\Phi_{II}(\rho, \xi) = \left(\frac{2q_t^2 S^2}{\epsilon_L^2\sqrt{\epsilon_L}\lambda_{De}}\right) \times \int_0^1 J_0\left(\frac{K_\perp \rho}{\sqrt{\epsilon_L}\lambda_{De}}\right) \; Sin\left(\frac{K_\perp \xi}{\sqrt{\epsilon_L}\lambda_{De} M}\right) \; K_\perp \, dK_\perp. \tag{17}$$

Thus, we obtain for $\rho = 0$

$$\Phi_{II}(\xi) \approx \frac{2q_t^2 S^2 M}{\epsilon_L^2} \, \frac{Cos(\xi/L_s)}{|\xi|}, \tag{18}$$

where $L_s = M \, \sqrt{\epsilon_L} \, \lambda_{De}$ and $|\xi| \gg L_s$ is assumed.

The second part of the additional dynamical potential is

$$\Phi_{III}(\mathbf{r}, t) = \frac{q_t^2 \lambda_{De}^2 C_d^2}{2\pi^2} \int \frac{\delta(\omega - k_\parallel v_t)(\omega^2 - k^2 C_s^2)}{(1 + \epsilon_L k^2 \lambda_{De}^2)^2 \, D} \exp{(i\mathbf{k} \cdot \mathbf{r})} \;\, k^2 k_\perp dk_\perp d\theta dk_\parallel d\omega, \tag{19}$$

where $D = (\omega^2 - k^2 C_s^2)(\omega - k_\parallel V_0)^2 - \{\omega_{k1}^2 (\omega - k_\parallel V_0)^2 + \omega_{k2}^2 (\omega^2 - k^2 C_s^2)\}$.

Introducing the dimensionless variable $\mathbf{K} = \mathbf{k}\sqrt{\epsilon_L}\lambda_{De}$ and following the same procedure, we finally obtain

$$\Phi_{III}(\xi) \simeq \frac{2q_t^2}{\epsilon_L(1 - M^{-2})} \; \frac{Cos(|\xi|/L_s)}{|\xi|}. \tag{20}$$

Comparing Eqs. (18) and (20), we note that for $S^2 \ll M$, $M > 1$

$$\Phi_{III}(\xi) \gg \Phi_{II}(\xi). \tag{21}$$

In order to have some appreciation of our theoretical result, we take parameters of a moderately doped n-$InSb$ where $n_{e0} = 10^{14}$ cm^{-3}, $T_e = 300^o$ K, $\epsilon_L = 15.8$, and $C_s \simeq 10^6$ cm/s. For $M = 1.5$, λ_{De} and L_s turn out to be $0.1\,\mu$m and $0.6\,\mu$m, respectively. This one-dimensional alignment of colloidal particles would modify the properties of the bulk semiconductor.

M. Salimullah, P. K. Shukla, S. K. Ghosh, H. Nitta, and Y. Hayashi, *J. Phys. D : Appl. Phys.* **36**, *958 (2003).*

M. Salimullah, A. M. Rizwan, S. K. Ghosh, P. K. Shukla, M. Nambu, H. Nitta, and Y. Hayashi, *J. Appl. Phys.* **97**, *124505 (2005).*