HIGHER ORDER THERMAL CORRECTIONS TO PHOTON SELF ENERGY

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Outline

- Relevance
- Finite Temperature Effects
- One Loop Corrections
- Higher Order Corrections
- Results

Relevance in Physics

- Particle Physics
- Nuclear Physics
- Condensed Matter Physics
- Plasma Physics

Relevance of Finite Temperature in Particle Physics

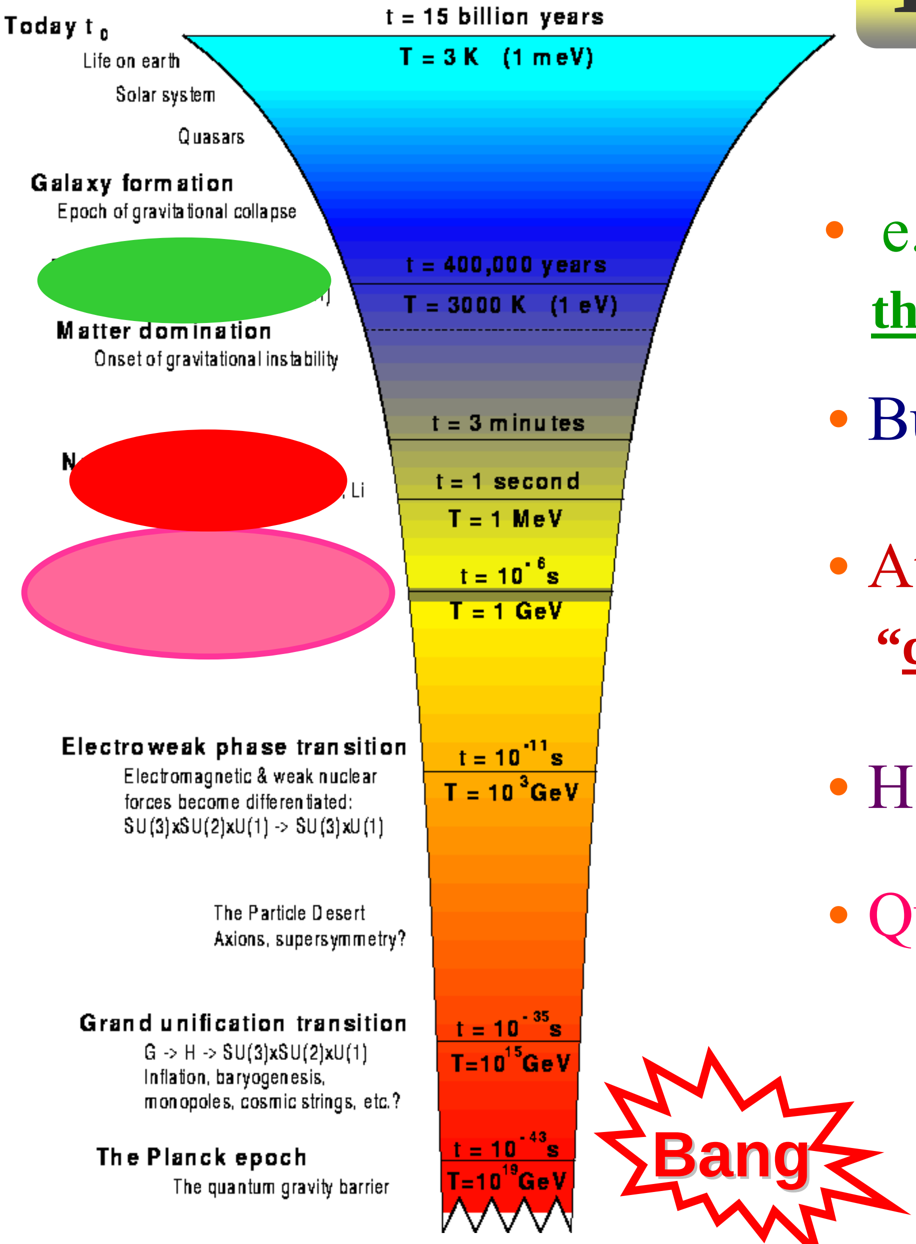
- Early Universe
- Quark Gluon Plasma
- High Energy Nucleus-Nucleus Collisions
- Astrophysics
- Cosmology

The Early Universe

- When the Big Bang occurred, the Universe was impossibly hot and dense. It rapidly expanded and cooled. A few interesting points in our context:
- *t* = 200sec, *T* = 1 billion degrees K

  It is cool enough for neutrons and protons to combine to form Deuterium, then Helium and traces of Lithium (**primordial nucleosynthesis**).
- For the next few 100,000 years it is too hot for electrons to form atoms. The Universe is filled with a hot **plasma** of electrons and nuclei, bathed in photons constantly interacting with both (like the interior of a star).

Big Bang



- e. m. decouple (T~ 1eV , t ~ $3{\cdot}10^5$ ys)

  thermal freeze-out

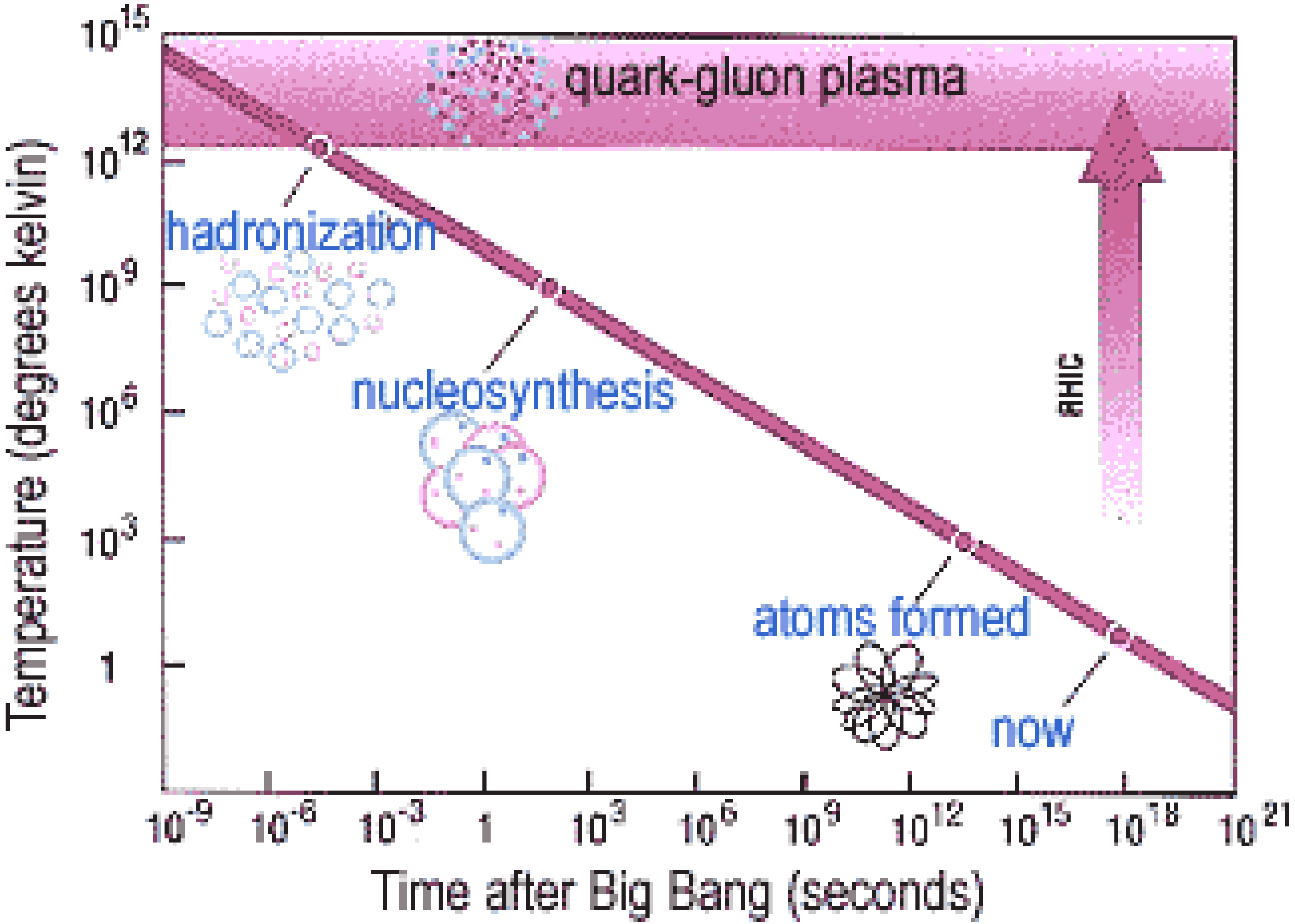
- But matter *opaque* to e.m. radiation

- Atomic nuclei (T~100 KeV, t ~200s)

  “**chemical freeze-out**”

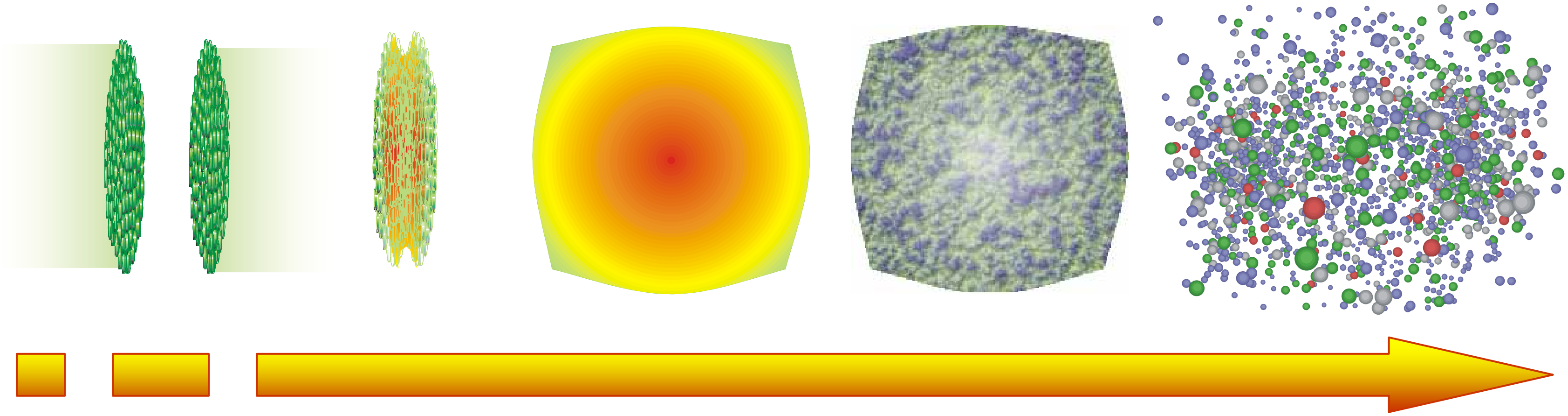
- Hadronization (T~ 0.2 GeV, t~ 10^{-2}s)

- Quark and gluons



quark-gluon plasma
hadronization
nucleosynthesis
atoms formed
now
RHIC
Temperature (degrees kelvin)
10^15
10^12
10^9
10^6
10^3
1
Time after Big Bang (seconds)
10^-9
10^-6
10^-3
1
10^3
10^6
10^9
10^12
10^15
10^18
10^21

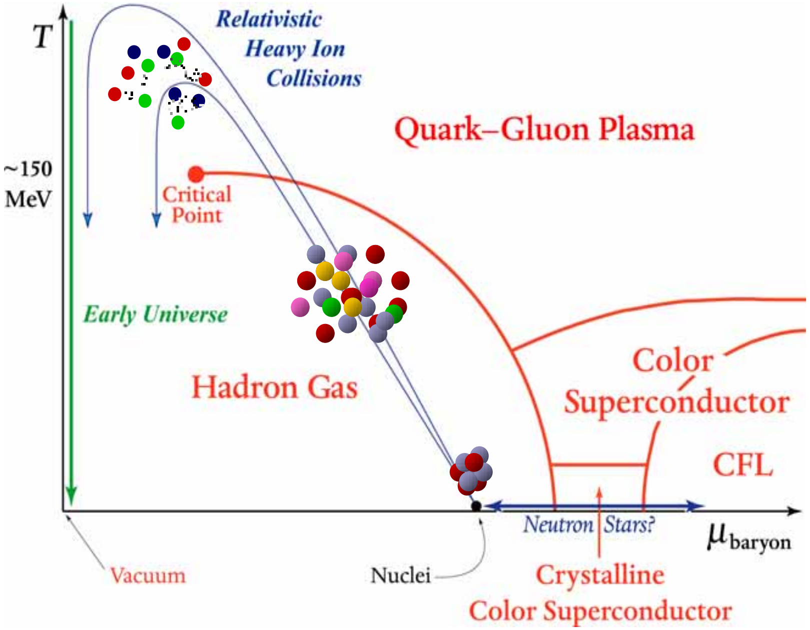
Heavy-ion collisions



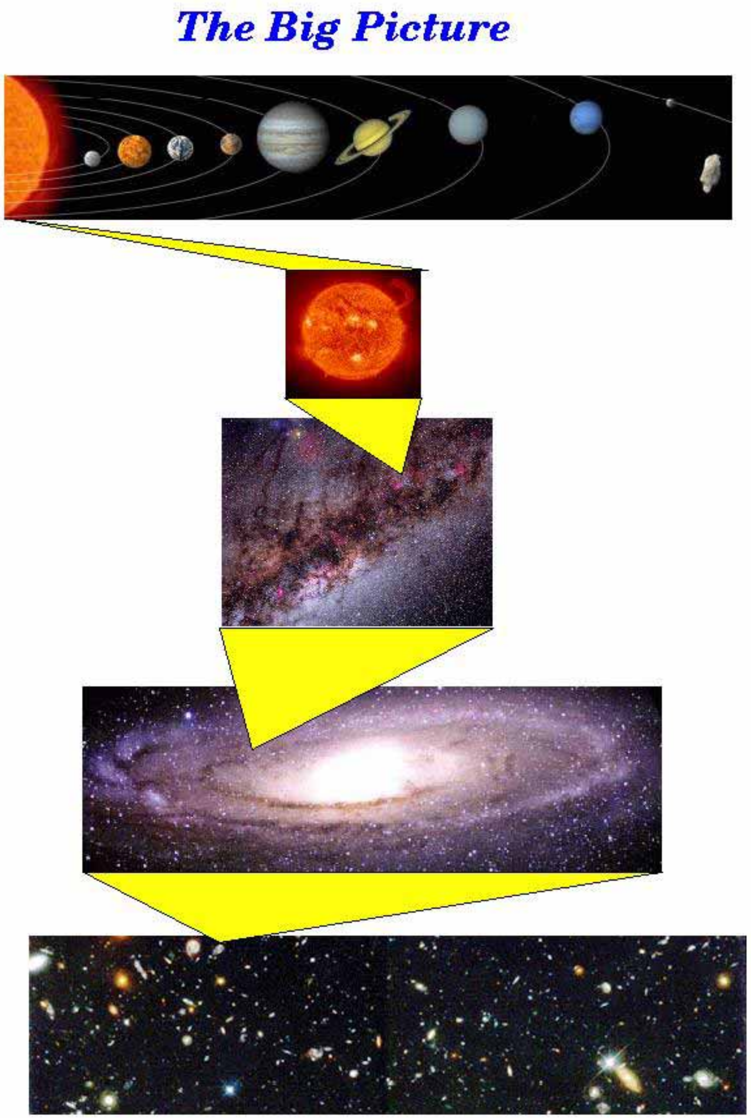
Time

quark-gluon plasma?

Energy

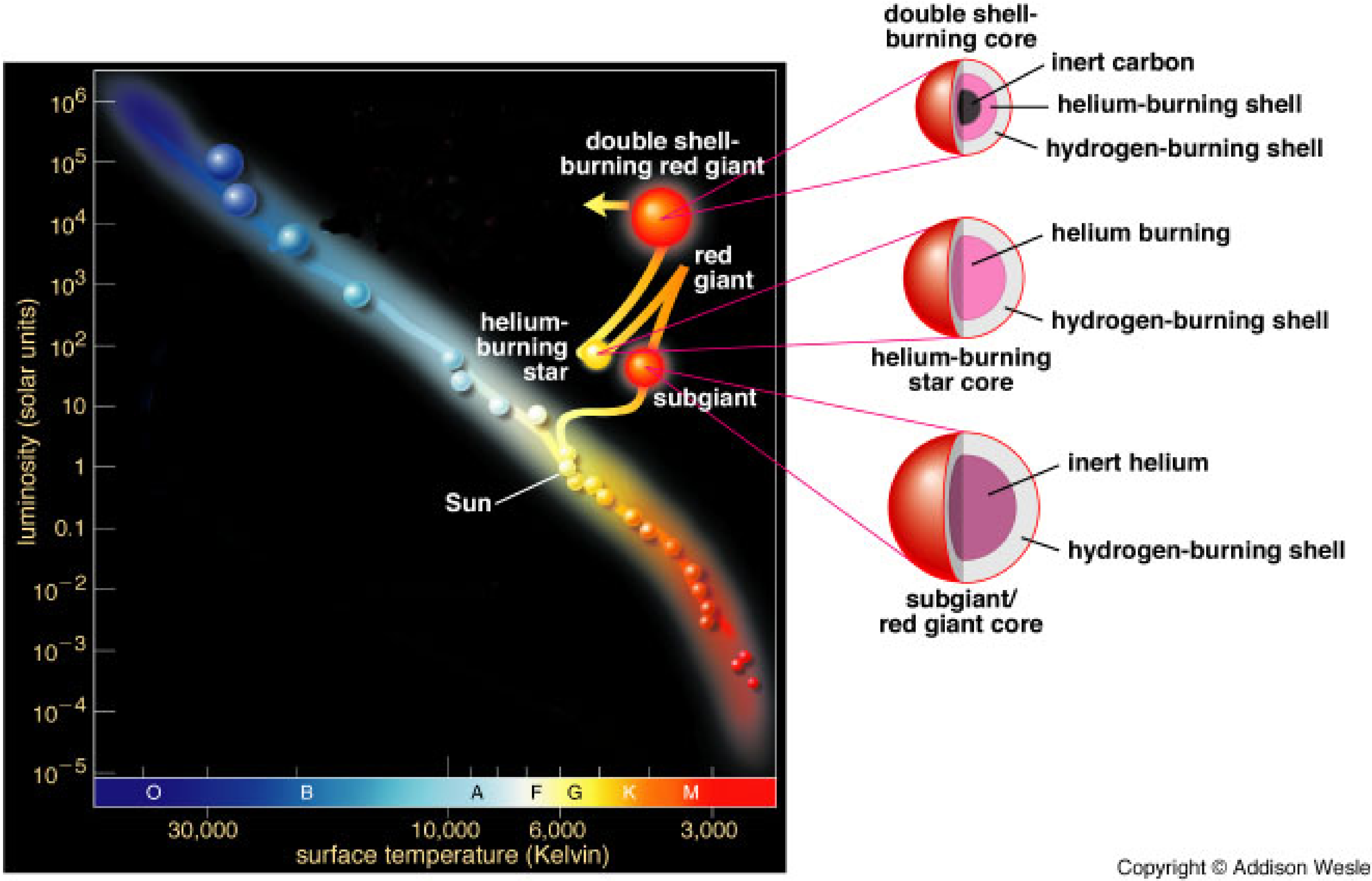

T
~150
MeV
Relativistic
Heavy Ion
Collisions
Quark–Gluon Plasma
Critical
Point
Early Universe
Hadron Gas
Color
Superconductor
CFL
Neutron Stars?
μ_{baryon}
Vacuum
Nuclei
Crystalline
Color Superconductor

Astrophysics



The Big Picture

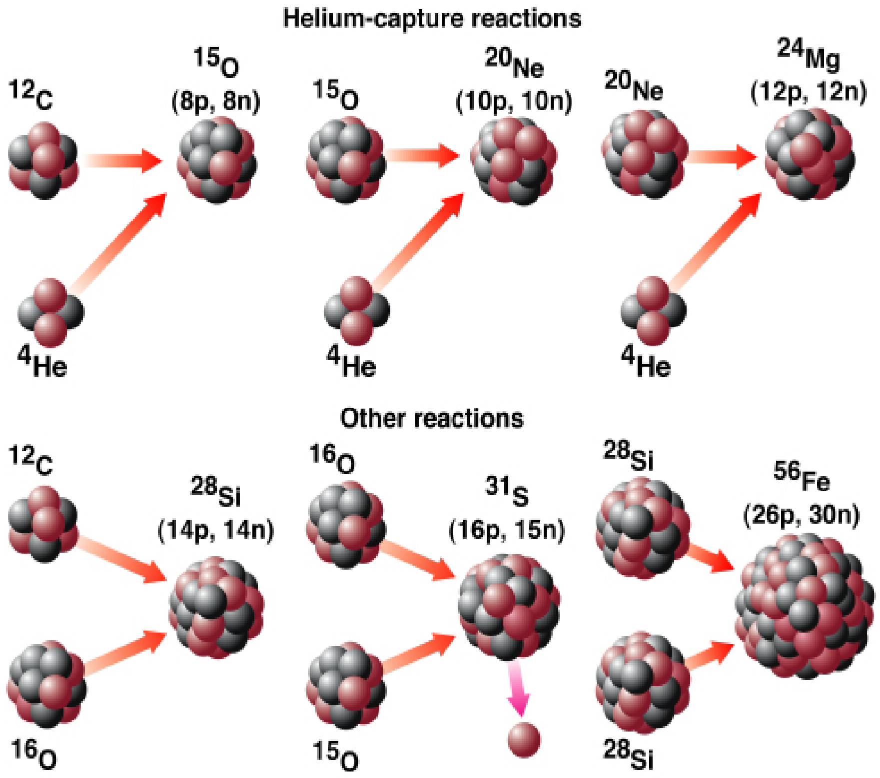
Planetary Nebulae



Supergiants

What happens to the high mass stars when they exhaust their He fuel?

- They have enough gravitational energy to heat up to 6 x 10^8 K.
  - C fuses into O
- C is exhausted, core collapses until O fuses.
- The cycle repeats itself.
  - O ⇨ Ne ⇨ Mg ⇨ Si ⇨ Fe



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A series of different types of fusion reactions in leading to luminous supergiants

- When helium fusion ceases in the core, gravitational compression increases the core's temperature above 600 million K at which carbon can fuse into neon and magnesium.

- When the core reaches 1.5 billion K, oxygen begins fusing into silicon, phosphorous, sulfur, and others.

- At 2.7 billion K, silicon begins fusing into iron.

- This process essentially stops with the creation of iron and a catastrophic implosion of the entire star initiates.

Supernova1006

- **Spectrum of limb**
  - keV photons

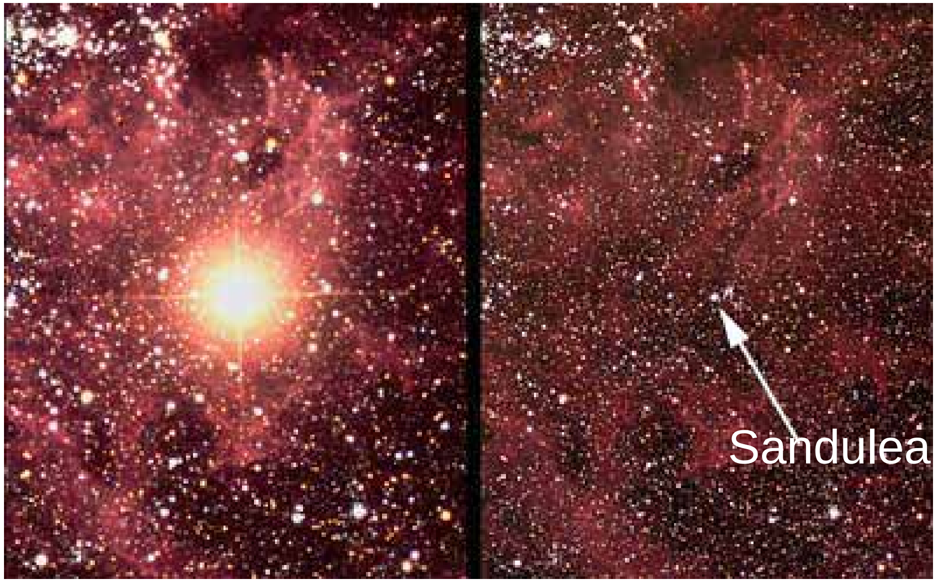
    $E_e \approx 100\text{TeV}$

- **Interior of SNR shows thermal ejecta**
  - showing outer regions of explosive nucleosynthesis

Supernova 1987A

Supernova have been important historically. Tycho and Kepler both observed supernova. The only supernova in modern time, visible to the naked eye, was detected on Feb. 23, 1987 and is known as SN1987A.

A tremendous amount of energy is released in a supernova. SN1987A emitted more than 100 billion times as much visible light as the Sun for over one month! Temperatures as high as 2×10^{11} K were reached.



Images of the star Sanduleak before and after it went supernova.

Something to think about: Sanduleak is 169,000 light-years from Earth. This means that SN1987A actually occurred in 167,000 BC.

Finite Temperature Effects

- In quantum field theory, the thermal background effects are incorporated through the radiative corrections.

- The renormalization of electric charge through the photon self-energy in different frameworks has been of interest in literature.

- In finite temperature electrodynamics electric fields are screened due to the interaction of the photon with the thermal background of charged particles.

- The physical processes take place in a heat bath comprising of hot particles and antiparticles instead of vacuum.

- The statistical effects are calculated either in Euclidean or Minkowski space by using imaginary or real-time formalism, respectively.

- In Euclidean space the covariance breaks and time is included as an imaginary parameter.

- In the real-time formalism, an analytical continuation of the energies along with Wick's rotation restores covariance in Minkowski space at the expense of Lorentz invariance.

- The breaking of Lorentz invariance leads to the non-commutative nature of the gauge theories[1].

- The covariance is incorporated through the 4-component velocity of the background heat bath as

$$u^{\mu} = (1,0,0,0)$$

[1] C. Brouder, A. Frabetti hep-ph//0011161 and F. T. Brandt, Ashok Das, J. Frenkel, Phys. Rev. D65 (2002) 085017

Fermion propagator

- The tree level fermion propagator in momentum space is[2]

$$S_\beta(p) = (\not{p} - m)\left\{\frac{i}{p^2 - m^2 + i\varepsilon} - 2\pi\delta(p^2 - m^2)n_F(E_p)\right\},$$

where

$$n_F(E_p) = \frac{1}{e^{\beta(p.u)} + 1},$$

is the Fermi Dirac distribution function with $\beta = \frac{1}{T}$

[2] J. F. Donoghue and B. R. Holstein, Phys. Rev. D28 (1983) 340

Boson propagator

- The bosons are represented as

$$D_{\beta}^{\mu\nu}(p) = [\frac{i}{k^2 + i\varepsilon} - 2\pi\delta(k^2)n_B(k)]$$

where

$$n_B(E_k) = \frac{1}{e^{\beta(k.u)} - 1},$$

is the Bose Einstein distribution function.

One loop corrections

- Feynman diagrams are calculated in the usual way by substituting these propagators in place of the usual ones.
- The Lorentz invariance breaking and conserving terms remain separate at the one loop level since the propagators comprise of temperature dependent (hot) terms added to temperature independent (cold) terms.
- This effect has been studied in detail at the one-loop level[3].

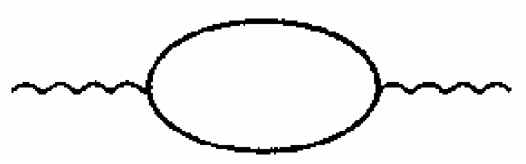


Fig.1

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[3] K. Ahmed and Samina S. Masood, Ann. Phys. (N.Y.) 207 (1991) 460

- The electric permittivity and the magnetic susceptibility of the medium are modified by incorporating the thermal background effects.
- At low temperatures, i.e., $T \ll m_e$ (m_e is the electron mass), the vacuum polarization tensor in order α does not acquire any hot corrections from the photons in the heat bath.
- This is because of the absence of self-interaction of photons in QED.

Two loop corrections

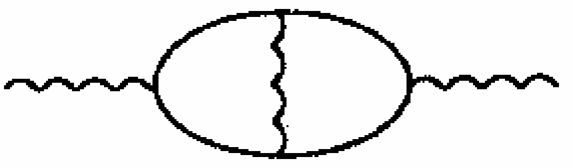

Fig. (2a)

Fig. (2b)

- At the higher-loop level, the loop integrals involve a combination of cold and hot terms which appear due to the overlapping propagator terms in the matrix element.

Higher order corrections

At the higher-loop level

- Specific techniques are needed (even at the two loop level) to solve them.
- Higher loops may not even be manageable analytically.
- The hot terms give the overlapping divergent terms.
- The removal of such divergences is already shown at the two loop level[4] for electron self energy.

Mahnaz Qader (Haseeb), Samina S. Masood, and K. Ahmed, Phys. Rev. D44 (1991) 3322; *ibid* Phys. Rev. D46, (1992) 5633

Low temperatures

- We restrict ourselves to the low-temperatures to prove the renormalizability of QED at the two-loop level through the order by order cancellation of singularities.

- The hot fermions contribution in background is suppressed and only the hot photons contribute from the background heat bath.

- The δ(0) type pinch singularities also appear in Minkowski space.

- This problem has been resolved in thermo field dynamics by doubling the degrees of freedom[5].

- The particle propagators become 2x2 matrices whose 1-1 elements correspond to the usual thermal propagators.

- We used an alternative technique to get rid of these δ(0) type pinch singularities through the identity[6]

$$i\pi[\delta(k^2)]^2 = -\frac{\delta(k^2)}{k^2 + i\varepsilon} - \frac{1}{2}\delta'(k^2).$$

[5] P. Landsman and Ch G. Weert, Phys. Rep. 145 (1987) 141

[6] L. R. Mohan, Phys. Rev. D14 (1976) 2670

- In this technique, the results depend on the order of doing the integration.

- The calculations are simplified if the temperature dependent integrations are performed before the temperature independent ones.

- The temperature independent loops can then be integrated using the standard techniques of Feynman parametrization and dimensional regularization as in vacuum.

- As an illustration of the difference of the order of integration, we compare the results of one of the terms.

- Consider the singular terms when the hot loop in Fig. (2b) is evaluated before the cold one, we simply get

$$g^{\mu\nu}\Pi^{a}_{\mu\nu}(p,T) = \frac{\alpha^2 T^2}{3}\left(1-\frac{2}{\varepsilon}\right),$$

- in the same term, the evaluation of the hot loop after the cold one gives

$$g^{\mu\nu}\Pi^{a}_{\mu\nu}(p,T) = -\frac{\alpha^2}{\pi^2}\frac{4\pi^2 T^2}{3\varepsilon} - \frac{\pi^2 T^2}{5} - \frac{2T^3}{5m^2}\zeta(3)\left(3|\mathbf{p}| + \frac{49}{3}p_0\right) + \frac{52 p_0^3 T^4}{5m^4|\mathbf{p}|}].$$

- The justification of this specific order is the fact that the temperature dependent part corresponds to the contribution of real background particles on mass-shell and incorporates thermal equilibrium.

- The breaking of Lorentz invariance changes these conditions for the cold integrals.

- The renormalization can only be proven with the preferred order of integration.

- At the higher loop level the vacuum polarization contribution is non zero, even at low temperature.
- The renormalization of the theory can only be proved if covariant hot integrals are evaluated before the cold divergent integrals.
- Once the hot loops are integrated out, the usual techniques can be applied to perform the cold loop integrations as in vacuum.

- At the two loop level, the vertex type corrections to the virtual electron in Fig. (2a) vanish and the self energy type corrections to the electron loop in Fig. (2b) contribute.

- The presence of the statistical effects of photons modifies the vacuum polarization and hence the electron charge.

- This leads to the changes in the electromagnetic properties of the hot medium even at low temperatures.

- The mass, the wave function, and the charge of electron have to be renormalized in the presence of the heat bath.
- We have calculated the corresponding renormalization constants in QED in the photon background up to the second order.

- The longitudinal and the transverse components of the vacuum polarization tensor are

$$\Pi_L(p,T) = -\frac{p^2}{|\mathbf{p}|^2} u^\mu u^\nu \Pi_{\mu\nu}(p,T) = \frac{2\alpha^2 T^2 p^2}{3|\mathbf{p}|^2}\left(1 + \frac{p_0^2}{2m^2}\right),$$

and

$$\Pi_T(p,T) = -\frac{1}{2}[\Pi_L(p,T) - g^{\mu\nu}\Pi^{a}_{\mu\nu}(p,T)] = \frac{\alpha^2 T^2}{3}\left[\frac{1}{2} - \frac{p^2}{|\mathbf{p}|^2}\left(1 + \frac{p_0^2}{2m^2}\right)\right]$$

respectively.

- These components of the vacuum polarization tensor can then be used to determine the electromagnetic properties of a medium with hot photons.

Results

- The change in electron mass due to temperature up to the order α^2 relative to the cold electron mass m is

$$\frac{\delta m}{m} = \frac{T^2}{m^2}\left(\alpha\pi + 4\alpha^2\right).$$

- The renormalizability of the self mass of electron is proven through the order by order cancellation of singularities at both loop levels.

- It can also be noted that the second order term is much smaller than the first order term.

- The charge renormalization constant of QED, the electron charge renormalization up to the order α^2 can be expressed as

$$Z_3 = 1 + \frac{\alpha^2 T^2}{6m^2}.$$

- The corresponding value of the QED coupling constant comes out to be

$$\alpha_R = \alpha(T = 0)(1 + \frac{\alpha^2 T^2}{6m^2}).$$

- The wave function renormalization constant comes out to be

$$Z_2^{-1} = 1 + \frac{\alpha}{4\pi}\left(4 - \frac{3}{\varepsilon}\right) - \frac{\alpha}{4\pi^2}\left(I_A - \frac{I^0}{E}\right) - \frac{\alpha^2}{4\pi^2}\left(3 + \frac{1}{\varepsilon}\right)I_A + \frac{2\alpha^2 T^2}{3\pi^2 m^2}.$$

- The results are an explicit proof of renormalizability of QED up to the two-loop level.

- They also estimate the temperature dependent modification in the electromagnetic properties of a medium.

- This helps to evaluate the decay rates and the scattering crossections of particles in such a media.

Applications

- Nucleosynthesis
- Leptogenesis
- Neutron stars
- Supernovae
- White Dwarfs