Exclusive B Decays In Universal Extra Dimension Model

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Outline

- Introduction to Extra Dimensions
- Universal Extra Dimension: ACD Model
- Rare *B* meson Decays in Universal Extra Dimension
- Conclusion.

Introduction to Extra dimension I

- Why Extra Dimensions?
  1. Quantization of gravitational interactions (String Theory).
  2. Addresses the hierarchy problem.
  3. Provide new dark matter candidates.
  4. ......
- There are several models of the extra dimensions.
  1. Large extra dimensions (Arkani-Hamed,Dimopoulous,Dvali)
  2. Wraped Extra dimensions (Randall, Sundrum)
  3. Univeral Extra dimensions (Applequest, Cheng, Dobrescu)

Universal Extra dimension Model I

- Among differnt models of the extra dimensions, which differ from one another depending on the number of extra dimensions, the most intresting one are the scenarios with universal extra dimensions (UED).
- In UED models all SM particles are allowed to propogate in the extra dimensions and compactification of an extra dimensions leads to the appearnce of Klauza-Klein (KK)partners of the SM fields in the four-dimensional description of the higher dimensional theory.
- The Applequist,Cheng and Dobrescu (ACD) model with one universal extra dimension is very attractive, because it has only one free parameter w.r.t SM, which is the inverse of compactification radius R.

Universal Extra dimension Model II

- By analyzing the signature of extra dimensions in differnt processes, one can get bounds on the size of the extra dimensions, which are differnt in differnt models.
- In case of UED model, bounds are more severe, and in the 5-d scenario constraints from Tevatron run I allow one to put the bound $1/R \geq 300$ GeV.

ACD Model I

- ACD model is the minimal extension of the SM in $4+\delta$-dimension.
- In literature the simplest case is $\delta = 1$ is considered.
- If an extra dimensions exist and is compactified,fields living in all dimensions would manifest themselves in the 3+1 dimension by the appearence of the KK excitations.
- Consider the 5-dimensional action for a (real) scalar field is given by

$$S_{5D} = \int d^4x \int dy(\partial_M \Phi \partial^M \Phi - M^2 \Phi\Phi) \tag{1}$$

ACD Model II

- Impose the boundary conditions on the field as well i.e . we require $\Phi(y = 2\pi R) = \Phi(y)$. Thus we can expand the 5-D scalar field as follows

$$\Phi = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{n=+\infty} \phi^{(n)}(x) e^{iny/R} \tag{2}$$

- Substituting the expansion of Φ into 5D action gives

$$S_{4D} = \int d^4x \sum_{n} \left[\partial_\mu \phi^{(n)} \partial^\mu \phi^{(n)} - (M^2 + \frac{n^2}{R^2}) \phi^{(n)} \phi^{(n)} \right] \tag{3}$$

- This is the $4D$ point of view the $5D$ scalar field appears as an (infinite) tower of $4D$ fields which are called the Kaluza-Klein (KK) modes.

ACD Model III

- The topology of the extra dimension is the orbifold S^1/Z^2, and the coordinate $x_5 = y$ runs from 0 to $2\pi R$.
- The **KK** modes of expansion of the fields are detemined from the boundary conditions at the two fixed points $y = 0$ and $y = \pi R$ on orbifold.
- One can rewrite the above mentioned **KK** decomposition in terms of functions which are even and odd under parity transformation P_5, i.e. $y \to -y$

$$\Phi(x,y) = \frac{1}{\sqrt{2\pi R}}\phi^{(0)} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[\phi_{+}^{(n)} \cos(\frac{ny}{R}) + \phi_{-}^{(n)} \sin(\frac{ny}{R})\right] \quad (4)$$

- The zero mode of the **KK** expansion corresponds to the ordinary SM fields.
- Even fields have corresponding ones in the four dimensional SM, whereas odd fields donot have corresponding ones in the SM.

B Decays in ACD Model I

- Flavor changing neutral current (FCNC) rare B decays opens a window to reveal new physics.
- These decays are not allowed at tree level, but they are induced at loop level and hence
  1. They are supressed in SM.
  2. Sensitive to the contribution of new particles circulating in the loops.
- The decay which I discuss here is $B \to K_1 l^+ l^-$
- At quark level the above mentioned decay can be written as $b \to s l^+ l^-$.

B Decays in ACD Model II

- The effective Hamiltonain for the decay $b \to sl^+l^-$ can be written as

$$H_{eff} = -4\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^{*}\sum_{i=1}^{10}C_i(\mu)O_i(\mu) \tag{5}$$

  where $O_i(\mu)$ is a local four quark operator and $C_i(\mu)$ are Wilson coefficents.

- One can write the above effective Hamiltonian in terms of the free quark decay amplitude

$$M(b \to sl^+l^-) = \frac{G_F\alpha}{\sqrt{2}}V_{tb}V_{ts}^{*} \times \left\{ \begin{array}{c} C_9^{eff}[\bar{s}\gamma^{\mu}Lb][\bar{l}\gamma^{\mu}l] \\ +C_{10}[\bar{s}\gamma^{\mu}Lb][\bar{l}\gamma^{\mu}\gamma^{5}l] \\ -2\hat{m}_b C_7^{eff}\left[\bar{s}i\sigma_{\mu\nu}\frac{\hat{q}^{\nu}}{\hat{s}}Rb\right]\left[\bar{l}\gamma^{\mu}l\right] \end{array} \right\} \tag{6}$$

B Decays in ACD Model III

- The amplitude given in above equation contains both long and short distance effects.
- The long distance effects encoded in the form factors and the short distance effects in the Wilson coefficents $C_i(\mu)$.
- Buras *etal* computed the Wilson co-efficents $C_i(\mu)$ at Next to leading order (**NLO**) in the **ACD** model.
- We use these results to study the decay $B \to K_1 l^+ l^-$.
- In **ACD** model, no new operators come's other than the SM.
- New Physics effects in **ACD** model will come only through the Wilson coefficent $C_i(\mu)$.
- In **ACD** model the modified Wilson coefficents can gets contribution from new particles.
- The new (**KK**) particles comes as an intermediate state in penguin and box diagrams.

B Decays in ACD Model IV

- The new Wilson coefficents can be expressed in terms of the functions $F(x_t, 1/R)$.
- The function $F(x_t, 1/R)$. can be written as follows

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n) \tag{7}$$

where $x_t = \frac{m_t^2}{M_W^2}$, $x_n = \frac{m_n^2}{M_W^2}$ and $m_n = \frac{n}{R}$

- The Rare decays in the **ACD** model considered here are governed as in the **SM** by various penguin (loop) diagrams.
- The **SM** contributions to flavor changing ($\Delta F = 1$) box diagrams are subleading and are not negligible.

B Decays in ACD Model V

- The relevant diagrams are Z^0 penguins, γ penguins, gluon penguins, γ magnetic penguins, chromomagnetic penguins and the corresponding functions are $C(x_t, 1/R)$, $D(x_t, 1/R)$, $E(x_t, 1/R)$, $D'(x_t, 1/R)$ and $E'(x_t, 1/R)$ respectively.
- The explicit form of these functions was discussed in A.J.Buras,M.Spranger,A.Weiler,Nucl.Phys.B 660,2005(2003)

Wilson Coefficents in ACD Model I

- The Wilson coefficents $C_7,\ C_9$ and C_{10} can be written in terms of the functions $C(x_t, 1/R)$, $D(x_t, 1/R)$, $E(x_t, 1/R)$, $D'(x_t, 1/R)$ and $E'(x_t, 1/R)$.
- In place of Wilson coefficent C_7, one defines an effective coefficent $C_7^{(0)eff}$ which is renormalization scheme independent

$$C_7^{(0)eff} = \eta^{\frac{16}{23}} C_7^{(0)}(\mu_W) + \frac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_8^{(0)}(\mu_W) + C_2^{(0)}(\mu_W) \sum_{i=1}^{8} h_i \eta^{\alpha_i}$$

where $\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}$ and

$$C_2^{(0)}(\mu_W) = 1,\ C_7^{(0)}(\mu_W) = -\frac{1}{2} D'(x_t, 1/R), \qquad (8)$$

Wilson Coefficents in ACD Model II

$$C_8^{(0)}(\mu_W) = -\frac{1}{2}E'(x_t, 1/R) \quad (9)$$

For C_9, in the ACD model and in the NDR scheme one has

$$C_9(\mu) = P_0^{NDR} + \frac{Y(x_t, 1/R)}{\sin^2\theta_W} - 4Z(x_t, 1/R) + P_E E(x_t, 1/R) \quad (10)$$

where $P_0^{NDR} = 2.60 \pm 0.25$ and the last term is numerically negligible, therefore

$$Y(x_t, 1/R) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n) \quad (11)$$

$$Z(x_t, 1/R) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n) \quad (12)$$

Wilson Coefficents in ACD Model III

- C_{10} is scale independent and is given by

$$C_{10} = -\frac{Y(x_t, 1/R)}{\sin^2\theta_W} \tag{13}$$

- The normalization scale is fixed at *b*-quark mass i.e. $\mu = \mu_b \simeq 5$ GeV.

Matrix element for the decay $B \to K_1 l^+ l^-$ I

- Exclusive decays $B \to K_1 l^+ l^-$ are described in terms of matrix elements of quark operators over meson states, which can be parametrized in terms of form factors.

$$
\begin{aligned}
\langle K_1(k,\varepsilon)\,|V_\mu|\,B(p)\rangle &= i\varepsilon_\mu^*(M_B+M_{K_1})V_1(s) \\
&\quad -(p+k)_\mu(\varepsilon^*\cdot q)\frac{V_2(s)}{M_B+M_{K_1}} \\
&\quad -q_\mu(\varepsilon^*\cdot q)\frac{2M_{K_1}}{s}\left[V_3(s)-V_0(s)\right] \\
\langle K_1(k,\varepsilon)\,|A_\mu|\,B(p)\rangle &= \frac{2\epsilon_{\mu\nu\alpha\beta}}{M_B+M_{K_1}}\varepsilon^{*\nu}p^\alpha k^\beta A(s)
\end{aligned}
$$

Matrix element for the decay $B \to K_1 l^+ l^-$ II

$$\langle K_1(k,\varepsilon)\,|\bar{s}i\sigma^{\mu\nu}q^{\nu}b|\,B(p)\rangle = \left[(M_B^2 - M_{K_1}^2)\varepsilon_\mu - (\varepsilon^*\cdot q)\,(p+k)_\mu\right]F_2(s) + (\varepsilon^*\cdot q)\left[q_\mu - \frac{s}{M_B^2 - M_{K_1}^2}(p+k)_\mu\right]F_3(s)$$

$$\left\langle K_1(k,\varepsilon)\left|\bar{s}i\sigma^{\mu\nu}q^{\nu}\gamma^5 b\right|B(p)\right\rangle = -i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha k^\beta F_1(s)$$

where $V_\mu = \bar{s}\gamma^\mu b$ and $A_\mu = \bar{s}\gamma^\mu\gamma^5 b$ are vector and axial vectors respectively,$\varepsilon^{*\mu}$ is the polarization vector for the final state axial vector meson.

- V_3 can be written as a combination of V_1 and V_2:

$$V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}}V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}}V_2(s)$$
$$V_3(0) = V_0$$

Matrix element for the decay $B \to K_1 l^+ l^-$ III

- Form factors are non-perturbative quantities and are scalar functions of the square of the momentum transfer.
- Differnt models are used to calculate these form factors.
- Form factors which we used for the analysis of branching ratio and forward backward asymmetry have been calculated using Ward identities.

Matrix element for the decay $B \to K_1 l^+ l^-$ IV

- Form factors which we used for the numerical analysis are given below

$$A(s) = \frac{A(0)}{\left(1 - s/M_B^2\right)\left(1 - s/M_B'^2\right)}$$

$$V_1(s) = \frac{V_1(0)}{\left(1 - s/M_{B_A^*}^2\right)\left(1 - s/M_{B_A^*}'^2\right)}\left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right)$$

$$V_2(s) = \frac{\tilde{V}_2(0)}{\left(1 - s/M_{B_A^*}^2\right)\left(1 - s/M_{B_A^*}'^2\right)} - \frac{2M_{K_1}}{M_B - M_{K_1}}\frac{V(0)}{\left(1 - s/M_B^2\right)\left(1 - s/M_B'^2\right)}$$

Branching ratio for the decay $B \to K_1\mu^+\mu^-$ I

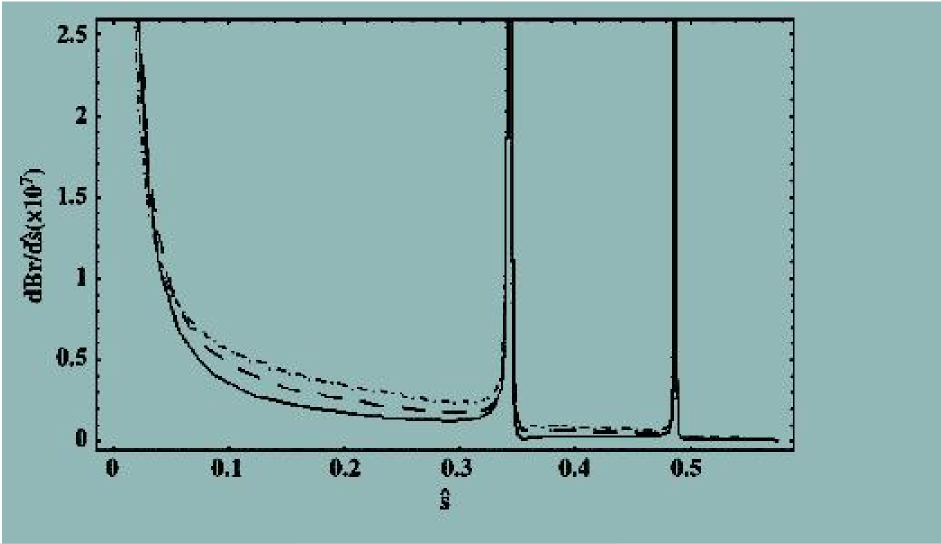
- By considering the final state lepton a muon, the branching ratio for $B \to K_1\mu^+\mu^-$ in the SM is

$$B(B \to K_1\mu^+\mu^-) = 0.72 \times 10^{-7}$$

- One can see from Fig (1) that there is a significant enhancement in the decay rate due to KK contribution for $1/R = 200$ GeV, whereas the value is shifted towards the SM at large values of $1/R$.
- The enhancement is prominant in low value of $\hat{s}$, but such effects are obscured by uncertainities involved in differnt parameters like the form factors, the CKM matrix elements etc. The numerical value at differnt values of $1/R$ is

$$B(B \to K_1\mu^+\mu^-) = 0.82 \times 10^{-7} \text{ for } 1/R = 200 \text{ GeV}$$
$$B(B \to K_1\mu^+\mu^-) = 0.75 \times 10^{-7} \text{ for } 1/R = 500 \text{ GeV}$$

Branching Ratio Results



Forward-backward asymmetry for the decay $B \to K_1 \mu^+ \mu^-$ I

- The effects of UED becomes more clearer if we look for the Forward-backward asymmetry(AFb) in the dilepton angular distribution, because it depends upon the Wilson coefficents.
- In the case of the decay $B \to K_1 l^+ l^-$ decay the investigation of the forward-backward asymmetry AFb in the dilepton angular distribution may aslo reveals effects beyond the SM.
- In the SM due to opposite signs of C_7 and C_9, AFb passes from its zero position.
- The zero position of AFb has a very weak dependence on the form factors.
- The zero position of AFb is also sensitive to compactification radius $1/R$.

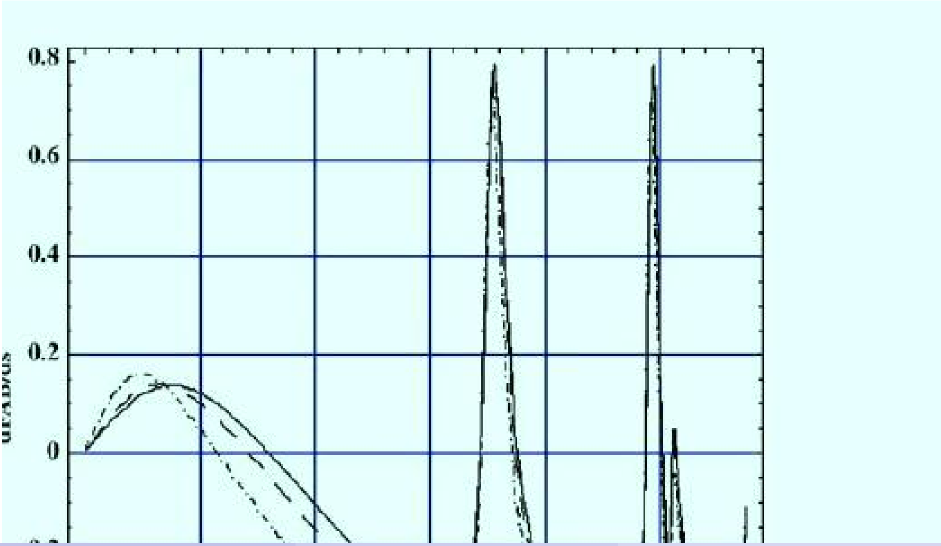
Forward-backward asymmetry for the decay $B \to K_1 \mu^+ \mu^-$ II

- The formula for the AFb is given below

$$A_{FB}(s) = \frac{\int_0^1 \frac{d^2\Gamma}{dsd\cos\theta_l} d\cos\theta_l - \int_0^1 \frac{d^2\Gamma}{dsd\cos\theta_l} d\cos\theta_l}{\int_0^1 \frac{d^2\Gamma}{dsd\cos\theta_l} d\cos\theta_l + \int_0^1 \frac{d^2\Gamma}{dsd\cos\theta_l} d\cos\theta_l} \qquad (14)$$

- Fig(2) shows the predictions of the zero position of AFb in SM, and in UED with $1/R = 200GeV$,$1/R = 500GeV$.
- As it is clear from fig(2) the zero position of AFb is also sensitive to compactification radius $1/R$.

Forward-backward Asymmetry Results



Conclusion

- We studied the semileptonic decay $B \to K_1\mu^+\mu^-$ in ACD model with single universal extra dimension.
- We studied the physical observables like branching ratio (*Br*) and forward-backward asymmetry(AFb), on the inverse compactifaction of radius $1/R$.
- The value of the *Br* is found to be larger than the SM value.
- The zero position of the AFb is very sensitive to $1/R$ and is shifted significantly towards left at $1/R = 200GeV$.
- The zero position of AFb approaches towards SM value if the value of $1/R$ gets increases.
- Future experiment in which more data are expected will put stringent constraints on the compactification radius and also give some deep understanding of $B - Physics$.