Ward Identities, $B \to \rho$ form factors and $|V_{ub}|$

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Motivation

- The semileptonic and radiative decays of heavy mesons play an outstanding role for determination of the parameters of standard model, in particular the quark mixing parameters.
- The theoretical understanding of weak decays of hadrons, and the measurements of the corresponding CKM matrix elements are consistently hampered by the presence of long distance QCD effects that are responsible for the binding of quarks into hadrons.
- These effects are hard to evaluate in a model independent way, and so tend to bring large uncertainities to the theoretical predictions for the weak decay amplitudes.
- They appear in the calculation of the matrix elements of the weak Hamiltonian operators, between the initial and final hadronic states.

Theoretical Approaches

- The theoretical understanding of exclusive decays is complicated mainly due to the fact that they involve hadronic form factors which are determined by nonperturbative contributions.
- These calculations of hadronic form factors for semileptonic B decays have been investigated by various theoretical approaches.
- Quark model
- Light-cone sum rules
- Heavy quark symmetry
- Dispersive bounds, and
- Experimentally constrained models.

Experimental facilities

[Upcoming and currently operating]

- BaBar at SLAC
- Belle at KEK
- LHCB at CERN
- B-TeV at Fermi Lab
- Planned τ-Charm factory at CLEO

Especially, a stringent test on the unitarity of the CKM mixing matrix elements in the SM will be made by these facilities. Accurate analysis of exclusive semileptonic B decays are thus strongly demanded for such precision tests.

Why choose $B \to \rho$?

- The future for the exclusive determinations of $|V_{ub}|$ appears promising.
- $B \to \pi l \nu$ appears to be a golden mode for future precise determination of $|V_{ub}|$.
- Although, the $B \to \rho l \nu$ mode will be more problematic for high precision because the broad width (of ρ) introduces both experimental and theoretical difficulties, but it will give more information on the long distance contributions.

Importance and Necessary ingredients to calculate $|V_{ub}|$

- From exclusive channels, to extract $|V_{ub}|$, the form factors of the channel must be known.
- The form factor normalization dominates the uncertainty on $|V_{ub}|$.
- The q^2 dependence of the form factors, which is needed to determine the experimental efficiency, also contributes to the uncertainty but at much reduced level.
- $|V_{ub}|$, the smallest in CKM mixing matrix, plays a crucial role in the examination of the unitarity constraints.

Scheme of the problem

- Ward Identities provide the constraints
- The constraints provided by Ward Identities will determine in the language of dispersion relations for relevant form factor contributions from continum as well as resonances which include not only $J^P = 1^-$ and 1^+ but also their radially excited states.
- This also provide normalization of form factors in terms of a universal function $g_+(0)$.

- We determined the coupling constants of 1^- and 1^+ resonances in terms of $g_+(0)$.

Predictions of coupling constants

$$g_{B^* B \rho^-} = \frac{2\,(1.26 \pm 0.17)}{f_B} .$$

$$\frac{g_{B_A^* B \rho^-}}{f_{B_A^* B \rho^-}} = \frac{M_{B_A^*}^2 - \left(M_B^2 - M_V^2\right)}{2}$$

$$g_{B_A^* B \rho^-} = 2.66 \times f_{B_A^* B \rho^-} \, \mathrm{GeV}^2 .$$

- All the form factors are double pole
- Proportional to single form factor $g_+(0)$
- The value of $g_+(0)$ is taken from LCSR calculation 0.29 ± 0.04
- The 1^- pole is at 5.33 GeV and 1^+ pole is at 5.71 GeV
- The poles are very close to the edge of physical region
- This means that poles should indeed dominate the behavior of form factors near $q^2 = q^2_{max}$

Form factors

$$V\left(q^2\right) = \frac{V\left(0\right)}{\left(1-q^2/M_B^2\right)\left(1-q^2/M_B'^2\right)}$$

$$A_1\left(q^2\right) = \frac{A_1\left(0\right)}{\left(1-q^2/M_{B_A^*}^2\right)\left(1-q^2/M_{B_A^*}'^2\right)}\left(1-\frac{q^2}{M_B^2-M_V^2}\right)$$

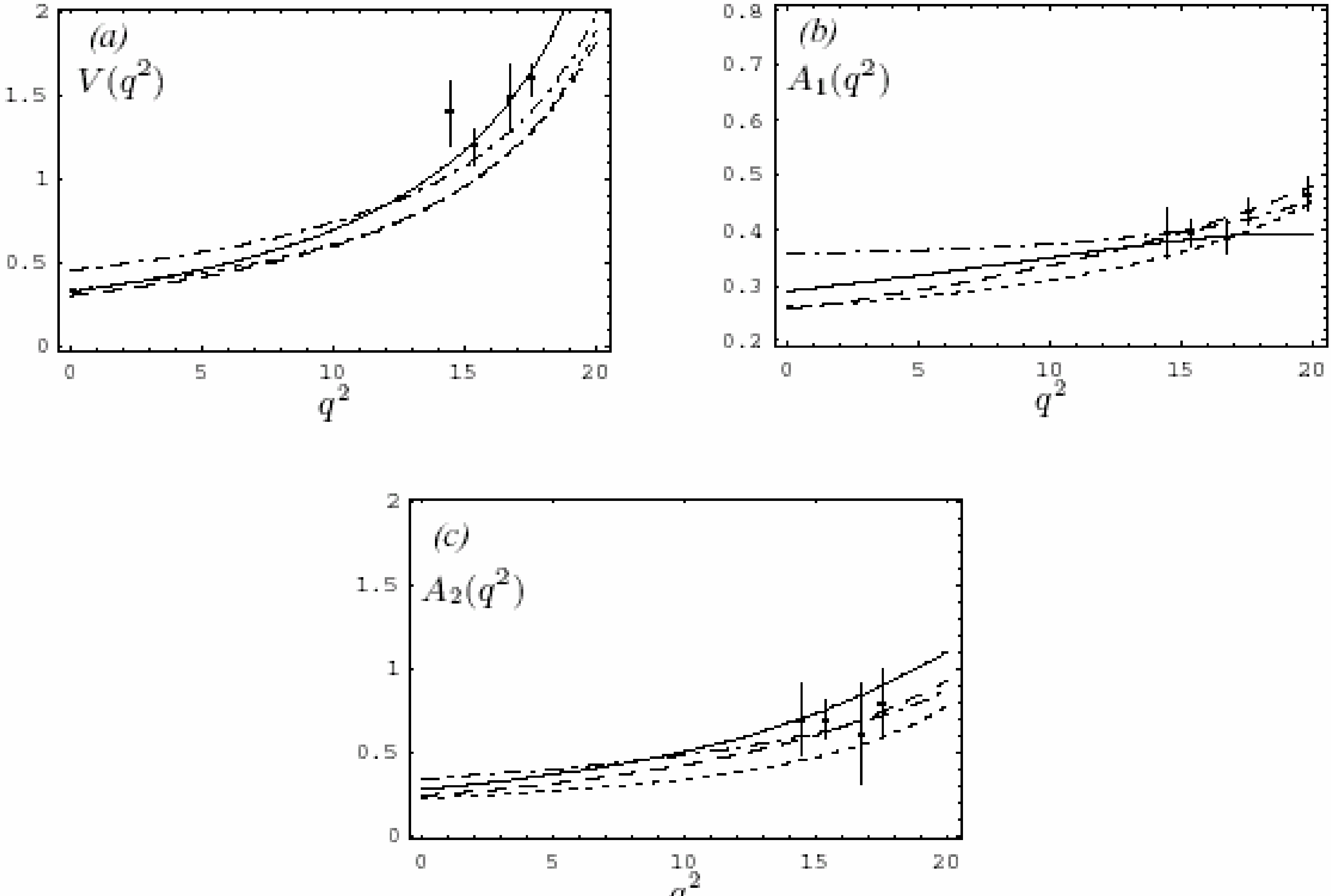
$$A_2\left(q^2\right) = \frac{\tilde{A}_2\left(0\right)}{\left(1-q^2/M_{B_A^*}^2\right)\left(1-q^2/M_{B_A^*}'^2\right)}$$

$$-\frac{2M_V}{M_B-M_V}\frac{A\left(0\right)}{\left(1-q^2/M_B^2\right)\left(1-q^2/M_B'^2\right)}$$

Values of form factors at q²=0

$$A_0(0) = 1.10 g_+(0) \simeq 0.320 \pm 0.044$$

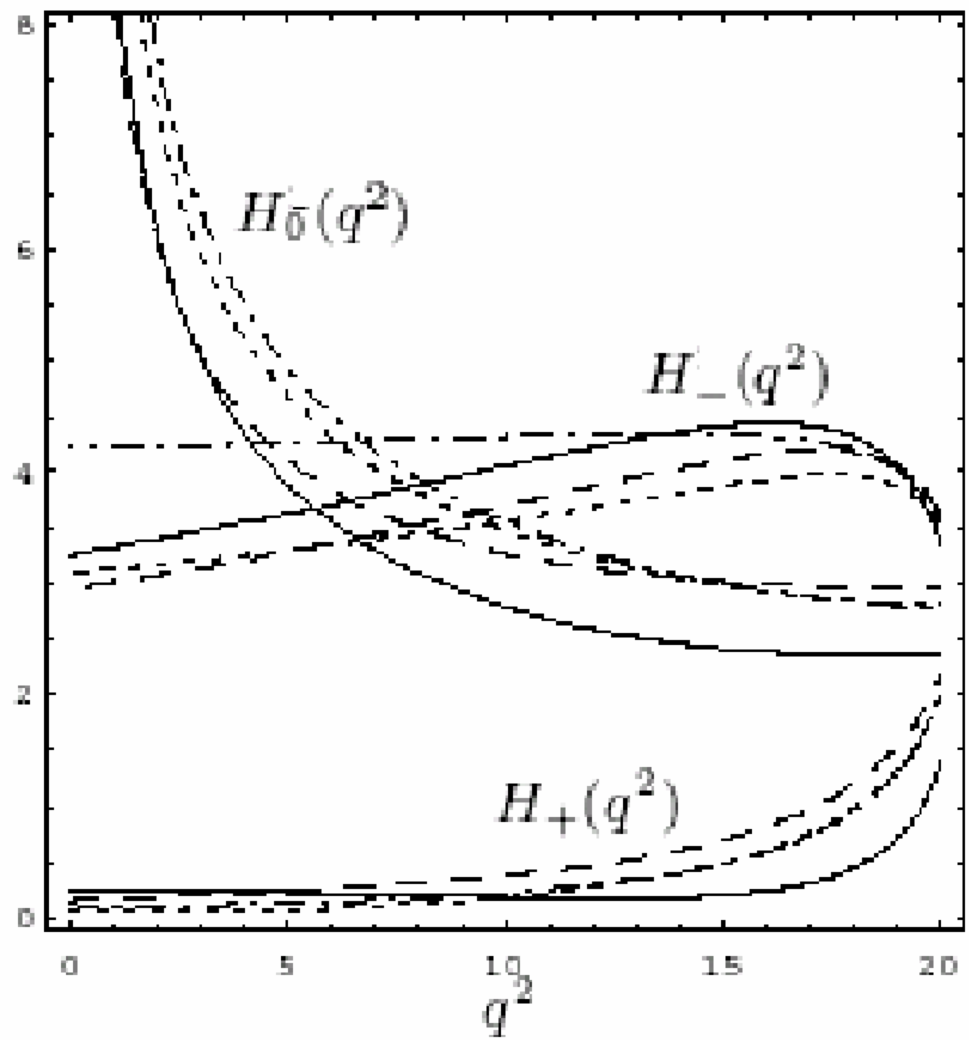
$$\begin{aligned} V(0) &= 0.332 \pm 0.046\,, \\ A_1(0) &= 0.29 \pm 0.04\,, \\ \tilde{A}_2(0) &= 0.389 \pm 0.054\,. \end{aligned}$$


(a)
$V(q^2)$
(b)
$A_1(q^2)$
(c)
$A_2(q^2)$
q^2

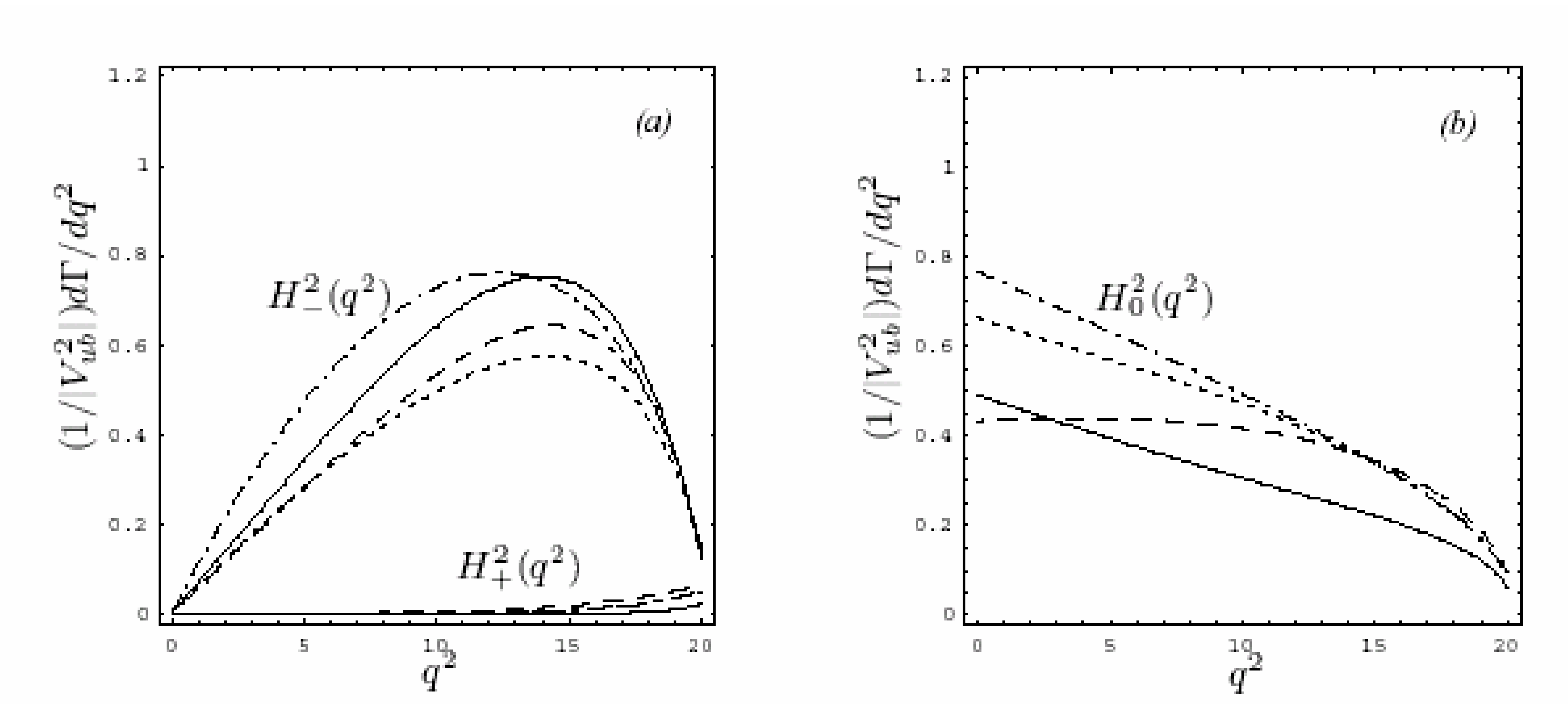
Helicity amplitudes

- Because the parent B-meson has spin zero, the vector meson-ρ and W* must have the same helicity.
- The amplitudes for the helicities 0, +1, -1 are proportional to H_0, H_+, H_-
- The dynamics of the hadronic current can be described by these helicity amplitudes
- The longitudinal part of the decay rate is proportional to H0
- The transverse part of the decay rate is proportional to H+ and H-
- The value of decay rate Γ+ is negligibly small while the value of Γ- is dominant

Helicity amplitudes



dΓ/dq² distribution for different helicity amplitudes is plotted and comparison with different approaches is given



Our observation agrees with the observation of

- Gilman and Singleton
- Korner and Schuler

Contradicts to the recent observation of

- Ali and Safir in the LEET approach

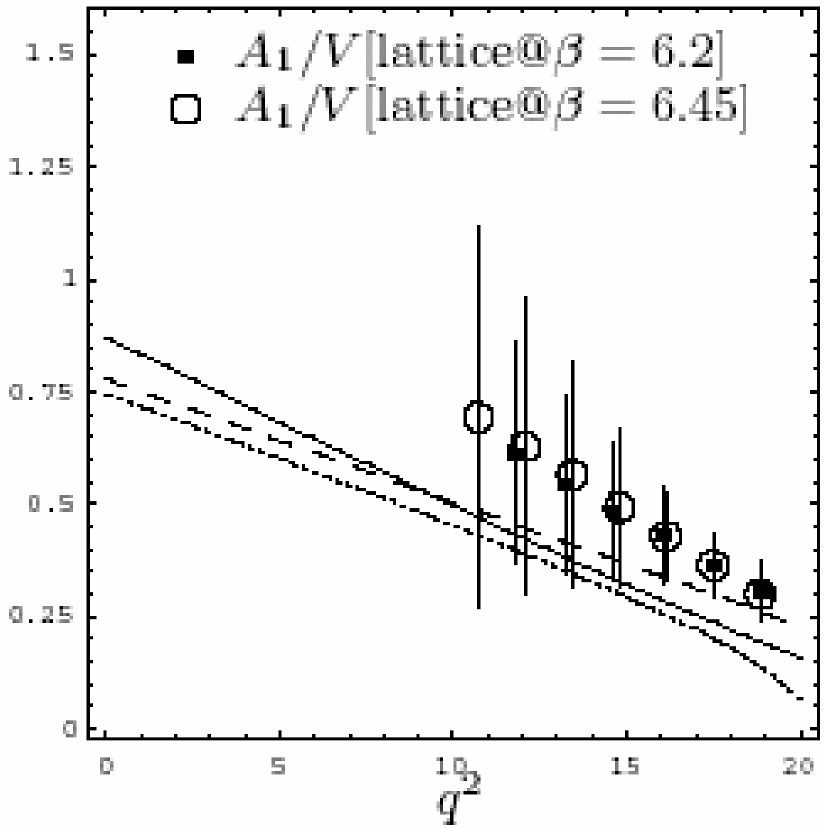
Longitudnal and transverse decay rates are calculated and their ratio is compared with different approaches

	Γ_-	Γ_+	$\Gamma_L = \Gamma_0$	Γ_T	Γ_{Total}	Γ_L/Γ_T
Our	9.662	0.059	6.095	9.721	15.82	0.63
QM [14]	-	-	-	-	15.8±2.3	0.88±0.08
LCSR [9]	-	-	-	-	13.5±4.0	0.52±0.08
Lattice [7]	-	-	-	-	$16.5^{+3.5}_{-2.3}$	$0.80^{+0.04}_{-0.03}$
ISGW2 [13]	-	-	-	-	14.2	0.3

Large energy limit gives the A_1 / V ratio

$$\frac{A_1\left(q^2\right)}{V\left(q^2\right)}=\frac{2E_V M_B}{\left(M_B+M_V\right)^2}=\frac{M_B^2+M_V^2-q^2}{\left(M_B+M_V\right)^2}$$

This is plotted by dashed line



Observable	V/A_1 $q^2=0$	V/A_1 $q^2 = 14.53\,\mathrm{GeV}^2$	V/A_1 $q^2 = 14.68\,\mathrm{GeV}^2$	Γ_+/Γ_-
Our	1.146	2.96	3.00	0.006
HQET-LEET [20]	1.29	2.63	2.66	0.02
Lattice [50]	-	2.05	2.02	-

Conclusion

- The form factors normalization is essentially determined by a single constant $g_+(0)$.
- A parameterization is used which take into account potential corrections to single pole dominance arising from radial excitations of M (B-meson mass) as suggested by dispersion relations.
- Predict the value of g_{B*Bρ} and relation between S-wave and D-wave couplings
- Our results of $|V_{ub}|$ agree with the experimental results
- The error in decay branching ratio is ~ 14%