$B \to \gamma\, l\, \nu$ *Decays*

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?

Standard Model process

$$B \to l\nu$$

- *Direct measurement of f_B*
- *CKM matrix element – V_{ub}*
- *New Physics beyond S.M.*

(at tree level)

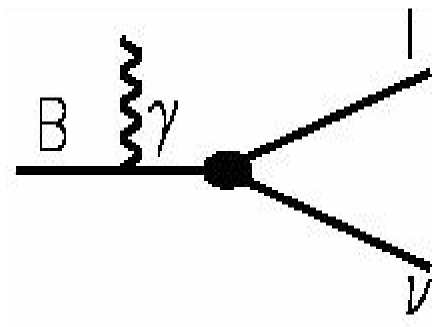
The decay width

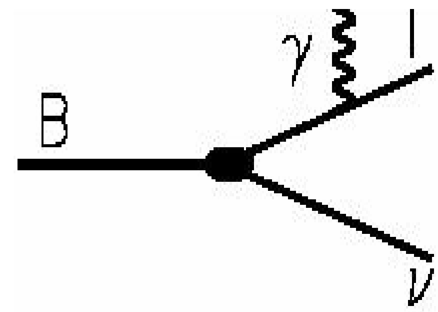
$$\Gamma\left(B \to l\nu\right) = \frac{G_F^2}{8\pi}\left|V_{ub}\right|^2 f_B^2 \frac{m_l^2}{M_B^2} M_B^3 \left(1 - \frac{m_l^2}{M_B^2}\right)^2$$

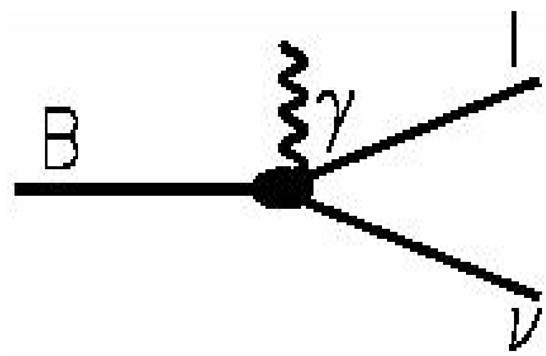
$$Br\left(B \to l\nu\right) \approx \begin{cases} 5.8\times 10^{-12} & \text{for } e^- \\ 2.2\times 10^{-7} & \text{for } \mu^- \end{cases}$$

[J. Lattery]

The Radiative Partner $B \to \gamma l \nu$







Amplitude:

- *Inner-bremstrahlung (IB)*

$$M_{IB} = ie\frac{G_F}{\sqrt{2}}V_{ub}f_B m_l \epsilon^*_\mu L^\mu$$

- *Structure Dependent (SD)*

$$M_{SD} = -i\frac{G_F}{\sqrt{2}}V_{ub}f_B m_l \epsilon^*_\mu H^{\mu\nu} l_\nu$$

where

$$
\begin{aligned}
L^{\mu} &= m_l \bar{u}(p_\nu)(1+\gamma_5)\left(\frac{2p^{\mu}}{2p\cdot k} - \frac{2p_l^{\mu} + \not{k}\gamma^{\mu}}{2p\cdot k}\right) v(p_l, s_l) \\
l^{\mu} &= \bar{u}(p_\nu)\gamma^{\mu}(1+\gamma_5) v(p_l, s_l), \\
H^{\mu\nu} &= iF_V\left(q^2\right)\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta - F_A\left(q^2\right)\left(k\cdot q g^{\mu\nu} - q^{\mu}k^{\nu}\right), \\
q^{\mu} &= (p-k)^{\mu} = (p_l + p_\nu)^{\mu}.
\end{aligned}
$$

And the decay constant and form factors are defined as

$$\left\langle 0 \left| \bar{u}\gamma^{\mu}\gamma_5 b \right| B(p) \right\rangle = -i f_B p^{\mu}$$

$$\left\langle \gamma(k) \left| \bar{u}\gamma^{\mu}\gamma_5 b \right| B(p) \right\rangle = \left[(\varepsilon^{*}.p^{\mu})k^{\mu} - \varepsilon^{*\mu}(p.k) \right] F_A(q^2)$$

$$\left\langle \gamma(k) \left| \bar{u}\gamma^{\mu} b \right| B(p) \right\rangle = -i\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\nu}^{*} p_{\alpha} k_{\beta} F_V(q^2)$$

Calculation of F_V and F_A

- Ward Identities
- Gauge Invariance
- Pole contributions
- Coupling Constants
- Branching Ratio

In order to obtain a universal normalization of these form factors at $q^2=0$, we define

$$
\begin{aligned}
\langle \gamma(k,\epsilon) | i\bar{u}\sigma_{\alpha\beta} b | B(p) \rangle = & -i\varepsilon_{\alpha\beta\rho\sigma}\epsilon^{*\rho}(k)\left[(p+k)^{\sigma} g_{+} + q^{\sigma} g_{-}\right] \\
& -iq\cdot\epsilon^{*}(k)\varepsilon_{\alpha\beta\rho\sigma}(p+k)^{\rho} q^{\sigma} h \\
& -i\left[q_{\alpha}\varepsilon_{\beta\rho\sigma\tau}\epsilon^{*\rho}(k)(p+k)^{\sigma} q^{\tau} - \alpha \leftrightarrow \beta\right] h_{1} \\
& -i\left[(p+k)_{\alpha}\varepsilon_{\beta\rho\sigma\tau}\epsilon^{*\rho}(k)(p+k)^{\sigma} q^{\tau} - \alpha \leftrightarrow \beta\right] h_{2}
\end{aligned}
$$

only h_1, g_- and h get pole contribution from $B^*(1^-)$ and $B_A^*(1^+)$ mesons

$$h_1\Big|_{pole} = -\frac{1}{2}\frac{g_{B^*B\gamma}}{M^2_{B^*}}\frac{f_T^{B^*}}{1-q^2/M^2_{B^*}} = \frac{R_V}{M^2_{B^*}}\frac{1}{1-q^2/M^2_{B^*}}$$

$$g_-\Big|_{pole} = -\frac{g_{B_A^*B\gamma}}{M^2_{B_A^*}}\frac{f_T^{B_A^*}}{1-q^2/M^2_{B_A^*}} = \frac{R_A^S}{M^2_{B_A^*}}\frac{1}{1-q^2/M^2_{B_A^*}}$$

$$h\Big|_{pole} = \frac{1}{2}\frac{f_{B_A^*B\gamma}}{M^2_{B_A^*}}\frac{f_T^{B_A^*}}{1-q^2/M^2_{B_A^*}} = \frac{R_A^D}{M^2_{B_A^*}}\frac{1}{1-q^2/M^2_{B_A^*}}$$

$$
\begin{aligned}
F_V\left(q^2\right) &= \frac{1}{m_b + m_q}\left\{g_+\left(q^2\right) - q^2 h_1\left(q^2\right)\right\} \\
&= \frac{1}{m_b + m_q}\left\{g_+\left(q^2\right) - R_V \frac{q^2}{M_{B^*}^2}\frac{1}{1 - q^2/M_{B^*}^2} - \sum_i \frac{q^2}{M_{B_i^*}^2}\frac{R_{V_i}}{1 - q^2/M_{B_i^*}^2}\right\} \quad (48) \\
F_A\left(q^2\right) &= \frac{1}{m_b - m_q}\left\{g_+\left(q^2\right) - q^2 h\left(q^2\right)\right\} \\
&= \frac{1}{m_b - m_q}\left\{g_+\left(q^2\right) - R_A^D \frac{q^2}{M_{B_A^*}^2}\frac{1}{1 - q^2/M_{B_A^*}^2} - \sum_i \frac{q^2}{M_{B_i^*}^2}\frac{R_{A_i}^D}{1 - q^2/M_{B_i^*}^2}\right\} \quad (49)
\end{aligned}
$$

coupling constants

$$\frac{R_A^S}{R_A^D} = -\frac{2g_{B_A^* B\gamma}}{f_{B_A^* B\gamma}} = -(M_B^2 - q^2)$$

$$g_{B^* B\gamma} = \frac{2g_+(0)}{f_B\left(1 - M_{B^*}^2 / M_{B^*}'^2\right)}$$

$$f_{B_A^* B\gamma} = \frac{M_{B_A^*}^2}{M_B} \frac{2g_+(0)}{f_{B_A^*}\left(1 - M_{B_A^*}^2 / M_{B_A^*}'^2\right)}$$

In HQET, the behavior of $g_+(q^2)$ near $q^2 = 0$ is known from LEET [J. Charles *et al*]

$$g_+\left(q^2\right) = \frac{\xi_\perp(0)}{\left(1 - q^2/M_{B^*}^2\right)^2}$$

$$g_+(0) = 0.29 \pm 0.04.$$

- do not hold for all q^2

parameterization, near $q^2 = 0$ and near the pole

$$F\left(q^2\right) = \frac{F(0)}{\left(1 - q^2/M^2\right)\left(1 - q^2/M'^2\right)}$$

takes into account

- potential corrections to single pole dominance, presumably arising from radial excitations of M.

- takes care of off-mass-shell-ness of couplings of B* or B^*_A with $B\gamma$ channel

The final expression for form factors

$$F_{V,A}\left(q^2\right) = \frac{F_{V,A}(0)}{\left(1 - q^2/M_B^2\right)\left(1 - q^2/M_B'^2\right)}$$

$$F_{V,A}(0) = \frac{2}{M_B} g_+(0)$$

And for the coupling constants

$$g_{B^*B\gamma} = \frac{2(1.25 \pm 0.17)}{f_B}$$

$$f_{B_A^*B\gamma} = \frac{2\,(8.83 \pm 1.22)}{f_{B_A^*}}$$

$$\begin{aligned} g_{B_A^*B\gamma} &= \frac{M_B^2 - M_{B_A^*}^2}{2} f_{B_A^*B\gamma} \\ &= -2.36 \times f_{B_A^*B\gamma} \end{aligned}$$

Branching Ratio

Using our form factors

$$Br\left(B \to \gamma l\, \nu\right) = 1.8 \times 10^{-6} \qquad \text{for } l=\mu$$

- CLEO $2\text{x}10^{-6}$
- Monte-Carlo Simulation $5.2\text{x}10^{-5}$
- Light-Cone QCD (2-5) $\text{x}10^{-6}$
- Bethe-Salpeter approach 0.9 $\text{x}10^{-6}$

Partial decay width Vs photon energy