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NEUTRINOS

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Certainly one of the most exciting areas of research at present is neutrino physics. Neutrinos are fantastically numerous in the universe and so to understand the universe we must understand neutrinos. "If there were no neutrinos, the sun and stars wouldn't shine. There would be no earth, no moon, no us. Without them we wouldn't be here". [Boris Kayser].

The energy of the sun is generated through the fusion

$$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e + 26.7\ MeV$$

- Neutrino was the first particle postulated by a theoretician:
- In 1930's protons, neutrons and electrons were considered as elementary particles. Such a picture was confronted with two fundamental problems:
- Conservation of energy and A.M. in β -decay

$$n \to p + e^{-}$$

- Continuous β spectrum can not be explained for 2-body final state if energy is conserved, since in that case E_e would have unique energy. Further final state would necessarily have integral A.M. while initial state has ½ integral A.M.

To solve these problems Pauli assumed that there exists a new electrically neutral elementary particle, with spin ½, mass less than electron mass and an interaction much weaker than photon interaction. Thus

$$n \longrightarrow p + e^{-} + \bar{\nu}_e$$

leading to continuous β spectrum and conservation of A.M.

Direct observation of $\bar{\nu}_e$ was made much later in 1950's. $\bar{\nu}_e$'s (electron-type antineutrino) are produced in the decay of pile neutrons in a fission reactor. These can be captured in hydrogen giving the reaction

$$\bar{\nu}_e + p \rightarrow n + e^-$$

whose cross-section was measured by Reines and Cowan:

$$\sigma_{exp} = (11 \pm 2.5) \times 10^{-44}\,\mathrm{cm}^2$$

Note the extreme smallness of the cross-section (nuclear cross sections are $\sim 10^{-24}\,\mathrm{cm}^2$). It is a reflection of the fact that neutrino has only weak interaction. It is remarkable that neutrinos which have almost no interaction with matter have contributed to some of the most important discoveries in Physics:

It is fair to say that the results of the last decade on neutrinos from the sun, from the atmospheric interaction of cosmic rays, and from reactors provide a compelling evidence that neutrinos have non-zero masses and mix. This is the first break with the standard model.

Neutrino Mass

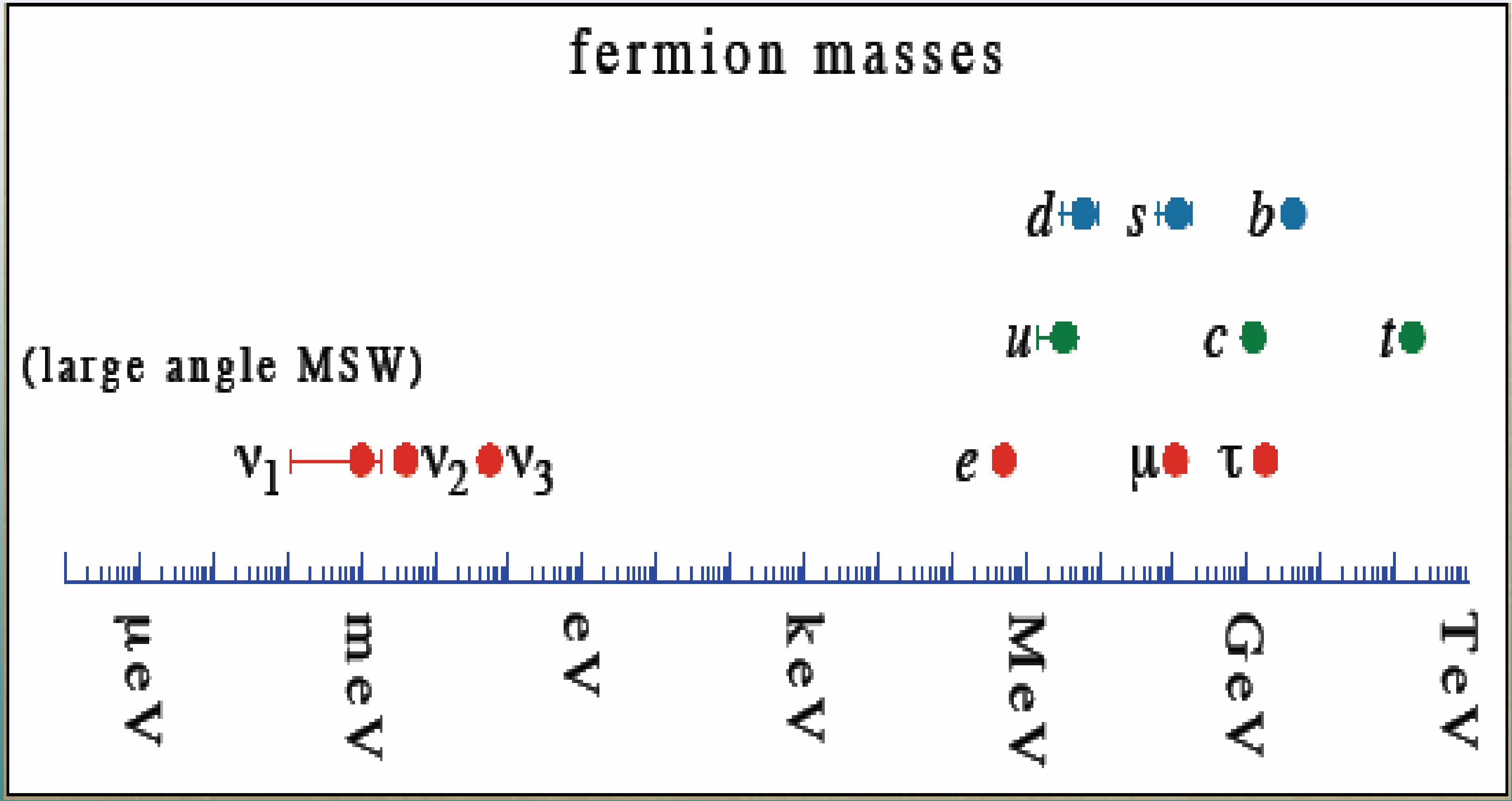
- Neutrino occurs in one helicity state (left handed). This together with lepton number conservation implies $m_\nu = 0$. However, there is no deep reason that it should be so. There is no local gauge symmetry and no massless gauge boson coupled to lepton number L, which therefore expected to be violated. Thus one may expect a finite mass for neutrino.

Moreover, all other known fermions, quarks and charged leptons, are massive. But the intriguing question is: why ?

$$m(\nu_e) \ll m(e)$$

Which needs to be understood, even though we do not understand why e.g. electron mass is what it is and why muons and tauons are heavier than electron.

Horizontal



fermion masses
d
s
b
u
c
t
(large angle MSW)
ν1
ν2
ν3
e
μ
τ
μeV
meV
eV
keV
MeV
GeV
TeV

Neutrino mass has added importance for two other reasons:

- The interesting phenomena of neutrino oscillations is possible if one or more of neutrinos have non vanishing mass.

- Non-vanishing of neutrino mass has important implication in Astrophysics and cosmology. It is a candidate for hot dark matter.

Astrophysical constraint on neutrino mass

The total mass-energy of the universe is composed of several constituents, each of which is characterized by its energy density, which is expressed in terms of critical density

$$\rho_{0i} \equiv \Omega_{oi}\rho_{c0}$$

Critical density is the minimum density required for the expansion of the Universe to be turned around by the gravitational attraction of all the matter in it and is defined as

$$\rho_{c0} = \frac{3H_0^2}{8\pi G_N} \approx 5.5 \times 10^3\, eV\ cm^{-3}$$

What is the neutrinos contribution to Hot Dark matter (since relic light ν's had relativistic velocity). Neutrinos are fantastically numerous

$$n_\nu = \frac{3}{10} n_\gamma = 112\, cm^{-3}$$

So if they have even a tiny mass, they can outweigh all the stars and galaxies in the universe.

$$\begin{aligned} \rho_{\nu 0} &= (112) \left(\sum_i m_{\nu i}\, eV \right) eV\, cm^{-3} \\ \Omega_\nu^{HDM} &= \frac{\rho_{\nu 0}}{\rho_{c0}} \end{aligned}$$

Unfortunately there is no direct particle physics evidence on $\Sigma_i m_{\nu i}$. We shall come back to this question later.

Here we simply note that

$$\rho_{\nu 0} \leq \rho_{c0}$$

$$\Rightarrow \Sigma_i m_{\nu i} \leq 49 \text{ eV}$$

This is the Astrophysical constraint on light neutrino masses.

Double β-Decay

The double β-decay is another way to look for a finite mass of neutrino. Two kinds of double β-decay can be considered:

$$(2\nu) \qquad (A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\nu_e$$

$$(0\nu) \qquad (A,Z) \rightarrow (A,Z+2) + 2e^{-}$$

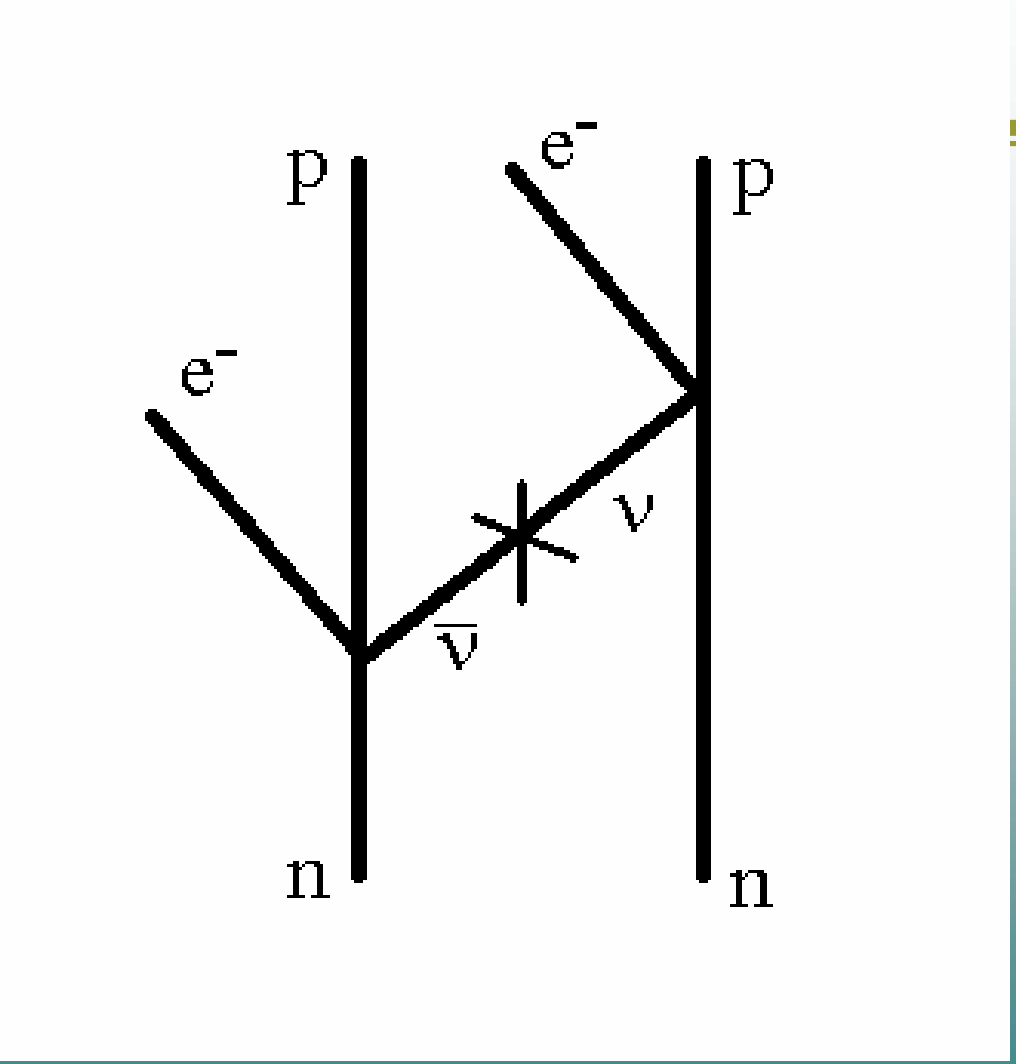
Usually the neutrinos are assumed to be Dirac particles.

Neutrino ν and antineutrino $\bar{\nu} \equiv \nu^{c}$

are distinct. In Majorana picture

ν and $\bar{\nu} \equiv \nu^{c}$

are identical.


p
e-
p
e-
ν
ν̄
n
n

The important physics issues in (0ν) double b-decay are:

- **Lepton number must not be conserved, which is possible if neutrinos are Majorana particles:**

$$\nu \equiv \bar{\nu}$$

- **Helicity of the neutrino cannot be exactly -1, this can be satisfied if $m_\nu \neq 0$.**

Thus (0ν) bb-decay is especially interesting

$$T_{1/2} \propto Q^{-5} \langle m_\nu \rangle^{-2},$$

Where decay Q value $\approx T_{e1} + T_{e2}$.

Here

$$< m_\nu > = \sum_i U_{ei}^2 m_{\nu i}$$

As shown the expectation value is weighted by neutrino's electron couplings.

There is direct evidence of $(2\nu)\beta\beta$ decays

$$(2\nu)\beta\beta \quad {}^{82}Se \rightarrow {}^{82}Kr$$

$$T_{1/2} = (1.1^{+0.8}_{-0.3}) \times 10^{20}\ yrs$$

For $(0\nu)\beta\beta$

$$(0\nu)\beta\beta \quad {}^{76}Ge \rightarrow {}^{76}Se + 2e^{-}$$

$$T_{1/2} \geq (1.9) \times 10^{25} yrs$$

One recent result, has claimed the evidence for this decay with the best value

$$T_{1/2} = (1.5) \times 10^{25} yrs$$

The analysis claims

$$\langle m_\nu \rangle = (0.39^{+0.17}_{-0.28})\ eV$$

M. V. Klapdu-Kleingroltrous et al., Mod. Phys. Lett. A16 (2001) 2409-2420.

If the above finding were to be confirmed, it would be the first indication of lepton number violation in nature and that Majorana neutrino can exist in nature.

We shall come to other implications of above value of $\langle m_\nu \rangle$ later.

Cosmological constraints on neutrino mass

Recent measurements of the fluctuations by an orbiting observatory called the Wilkinson Microwave Anisotropy Probe (WMAP) and their analysis have settled a number of issues about the universe, its age, its expansion rate and its composition.

Results:

$$H0 = (72 \pm 5)\ \text{km s}^{-1}\ \text{MPC}^{-1}$$

Age of universe = 13.4 ± 0.3 billion years.

$$\Omega = \rho/\rho_c = 1.02 \pm 0.02$$

$$\rho_{DM} = (1.26 \pm 0.21) \times 10^{-3}\ \text{eV/cm}^3$$

$$\Omega_{DM} = 0.23 \pm 0.05$$

$$\Omega_b = 0.046 \pm 0.005$$

$$\Omega_\nu < 0.015,\ \ m_\nu < 0.23\ \text{eV}$$

Note that the visible baryon density is only about 4.6%.

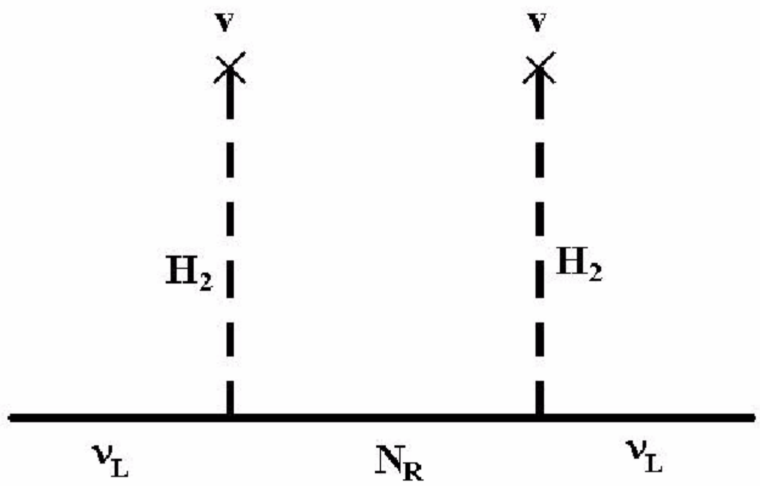
The Spectrum Scale of ν-mass from

		$\sim 0.2\, eV$
Tritium β decay	$\sum_i \left\|U_{ei}^2\right\| \left\|m_i\right\| \leq 2\, eV$	KATRIN
		Exp
Cosmology	$\sum_i m_i < 0.69\ eV$	$\sim\ (0.05 - 0.1)\, eV$
$\beta\beta 0\nu$	$< m_\nu \equiv \sum_i U_{ei}^2 m_i$	$\sim 0.02\, eV$
Claimed Observation	$\left(0.39^{+0.17}_{-0.28}\right)\ eV$	

Origin of Neutrino Masses

The minimal standard model involves 3 chiral neutrino states, but it does not admit renormalizabile interactions that can generate neutrino masses.

If one allows right-handed neutrinos νR which are SUL(2)×UY(1) singlets, then one can write Yukawa interactions.



$$L_x = \frac{1}{2}\begin{pmatrix} \bar{\nu}_L & \bar{N}_L^c \end{pmatrix}\begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}\begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + hc$$

$$\nu_L \longleftrightarrow \nu_R^c,\ N_R \longleftrightarrow N_L^c \ by\ CPT$$

$$\nu_L,\ N_R\ each\ have\ 3\ flavors$$

$$m_D\quad Dirac\ mass\ matrix\quad (Higgs\ Doublet)$$

$$M \equiv M^T\ Singlet\ Majorana\ mass\ matrix\ (Higgs\ singlet)$$

$In\ the\ ordinary\ seesaw\ eigenvalues \qquad M \gg m_D$

$$m_\nu^{eff} = -m_D M^{-1} m_D^T$$

$After\ the\ spontaneous\ symmtery\ breaking$

$the\ vacuum\ expectation\ value\ \langle\langle H_2\rangle = v = 175\ \rangle$

$generates\ the\ Dirac\ mass\ \ (m_D)_{ij} = h_{ij} v\ .$

Thus

$$m_{\nu\ell} \sim m_D^2 / M \approx v^2/M << m_\ell,$$

by requiring the existence of large scale M, associated with new physics. Indeed, since $v \approx 175$ GeV, $m_\nu \approx 0.03$ eV, for $M \approx 10^{15}$ GeV.

- Neutrino masses are a probe of physics at MGUT.
- Neutrino oscillations might remarkably provide a mechanism to measure extremely small masses and indirectly provide a new scale indicative of new physics.

Neutrino Oscillations

Oscillations in vacuum

- Neutrinos are produced in weak interactions as flavor eigenstates, characterized by e, μ, τ.

- The flavor eigenstates $|\nu_\alpha\rangle$ need not coincide with mass (energy) eigenstate $|\nu_i\rangle$ and are generally coherent superposition of such states

$$|\nu_\ell\rangle = \sum_i U_{\ell i}\,|\nu_i\rangle$$

where the mixing matrix is unitary. This matrix is characterized by 3 angles, $\theta_{12} = \theta_3$, $\theta_{13} = \theta_2$, $\theta_{23} = \theta_1$, one CP violating phase δ and two Majorana phases which we put equal to zero.

The probability at time t that ν_ℓ is converted into $\nu_{\ell'}$ is

$$P_{\nu_\ell \rightarrow \nu_{\ell'}} = \sin^2 2\theta \, \sin^2[1.27 \, \Delta m^2/E_\nu] \, L$$

Where L is the distance (measured in meters) travelled after ν_ℓ is converted into $\nu_{\ell'}$.

$\Delta m^2 = m_1^2 - m_2^2$ in units if eV^2 while E is measured in MeV.

Thus the oscillations in this simple case are characterized by the oscillation length

$$L_\nu = 4\pi \, E_\nu/\Delta m^2$$

And by the amplitude $\sin^2 2\theta$.

Oscillation in matter

- In traversing matter neutrinos intract with electrons and nucleus of intervening material and their forward scattering induces an effective potential energy

$$\mathrm{Sqrt}[2]\, G_F\, N_e,$$

so that the corresponding matter oscillation length is

$$L_0 = 2\pi/\mathrm{Sqrt}[2]\, G_F\, N_e = 1.7 \times 10 \;\; (\mathrm{m})/\rho(\mathrm{g/cm^3})Y_e$$

$$\frac{L_v}{L_0} = \frac{2\sqrt{2}G_F N_e E_\nu}{\Delta m^2}$$

$$= 0.22 \left[\frac{E_\nu}{1\,MeV}\right] \left[\frac{\rho Y_e}{100g/cm^3}\right] \left[\frac{7 \times 10^{-5} eV^2}{\Delta m^2}\right]$$

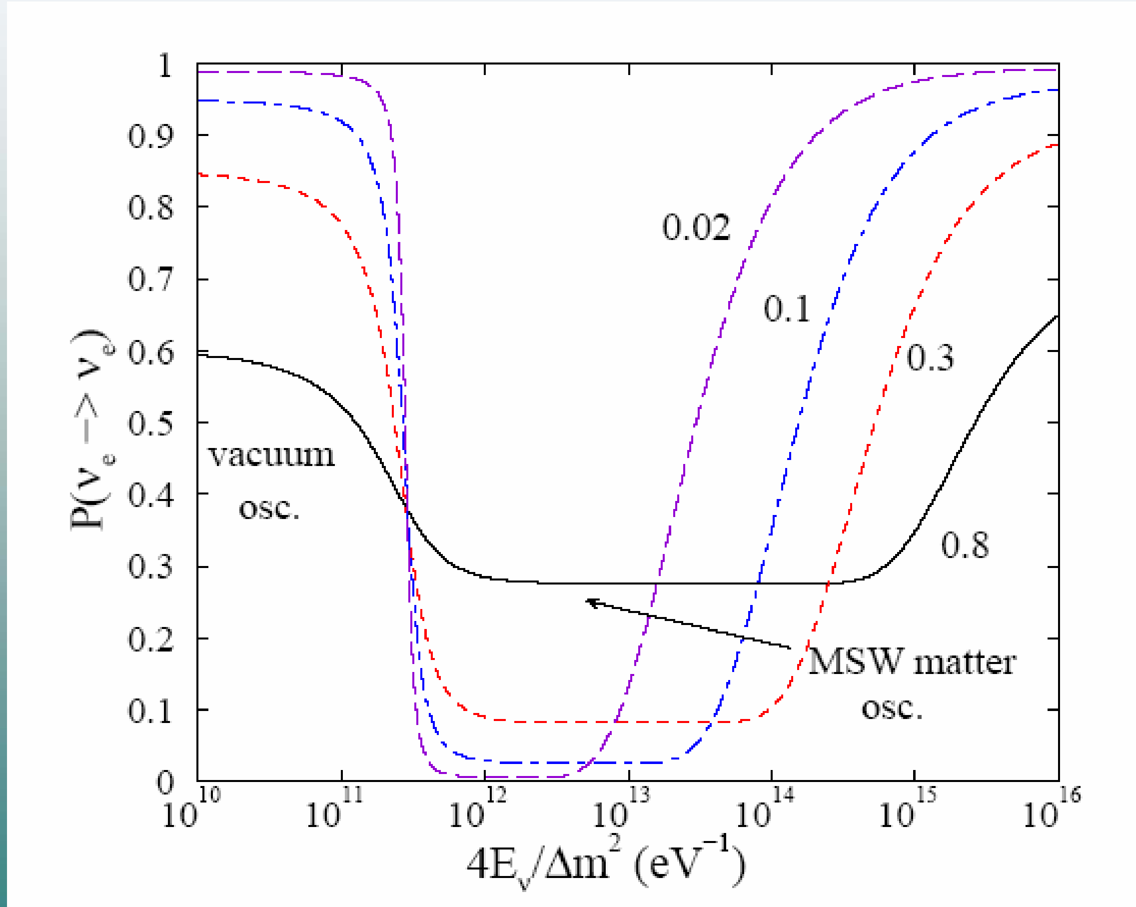
The transition point between the regime of vacuum and matter oscillations is determined by the ratio L_v / L_o

If it is greater than 1, matter oscillations dominate

If it is less than cos 2θ, vacuum oscillations dominate

Generally there is a smooth transition between these two regimes.

The survival probability $\langle P(\nu_e \to \nu_e)\rangle$ as a function of $E\nu$ is displayed for various mixing angles in figure.



Evidence for Oscillations

One looks for oscillations in two types of experiments:

(i) Appearance experiments:

Here one searches for a new neutrino flavor, absent in the initial beam, which can arise from oscillations.

- Atmospheric neutrino anomaly:

  Atmospheric neutrinos are produced in decays of pions (kaons) that are produced in the interaction of cosmic rays with the atmosphere:

$$p + A \rightarrow \pi^{\pm} + A'$$

$$\pi^{\pm} \rightarrow \mu^{\pm} \nu_\mu \; (\bar{\nu}_\mu)$$

$$\rightarrow e^{\pm} \nu_e \; (\bar{\nu}_e) \; \bar{\nu}_\mu \; (\nu_\mu)$$

These neutinos are detected in and beneath underground detectors through the reactions

$$\nu_\mu + n \to \mu^- + p,$$

$$\nu_\mu + p \to \mu^+ + n,$$

and

$$\bar{\nu}_e + n \to e^- + p,$$

$$\bar{\nu}_e + p \to e^+ + n,$$

And are respectively called μ-like and *e*-like events. The observed ratios of these events was found to be substantially reduced from the expected value ~ 2.

- There is compelling evidence that atmospheric neutrinos change flavor as the Super-Kamiokande experiment clearly indicated a deficit of up-ward μ-like events (produced about 10^4 km away at the opposite side of earth) relative to the downward going events (produced about 20 km above). The e-like events showed a normal zenth angle dependence.

- The data is described by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. The conversion probability $P_{\nu_\mu \leftrightarrow \nu_\tau}$ fits the data quite well for

$$\Delta m_{23}^2 = 2.0 \times 10^{-3} \quad eV^2, \quad \sin^2 2\theta_{23} \approx 1.0$$

Solar Neutrinos

Particularly compelling evidence that the solar neutrinos change flavor has been reported by the Sudbury Neutrino Observatory (SNO). SNO measures the high energy part of the solar neutrino flux (^{8}B neutrinos)

The reactions

$\nu d \to \nu n p$

$\quad \to e p p$

$\nu e \to \nu e$

employed by SNO. SNO measured arriving $\nu_e+\nu_\mu+\nu_\tau$ flux, $\phi_e+\phi_{\mu\tau}$, and the ν_e flux, ϕ_e. From the observed rates for the first two reactions, which involve respectively neutral current and charge current, SNO finds that the ratio of the two fluxes is

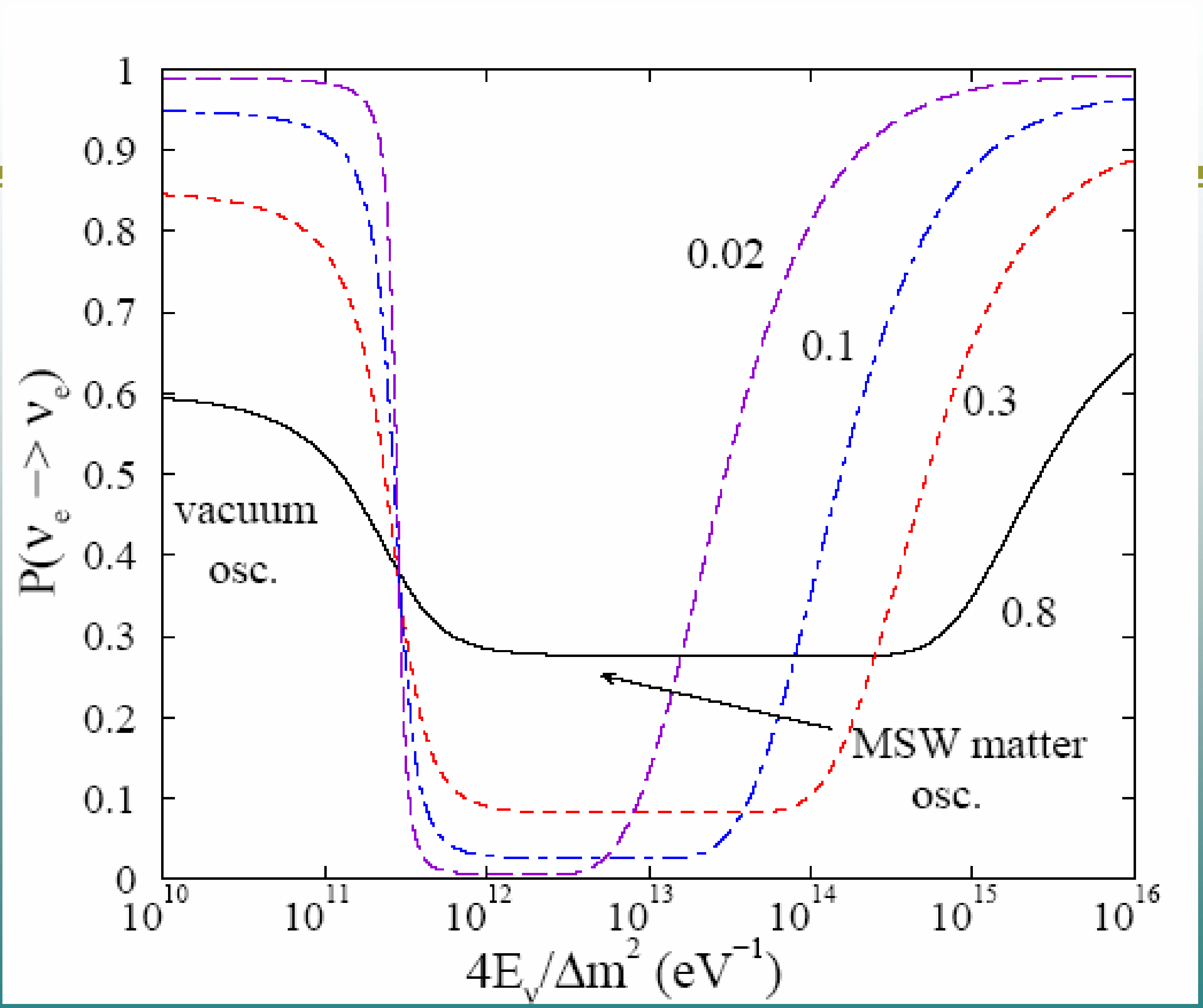
$$\frac{\phi_e}{\phi_e+\phi_{\mu\tau}} = 0.306 \pm 0.050.$$

This implies that the flux $\phi_{\mu\tau}$ is not zero.

Since all the neutrinos are born in nuclear reactions that produce only electron neutrinos, it is clear that neutrinos change flavor.

Corroborating information comes from the detection reaction $\nu e \rightarrow \nu e$, studied by both SNO and Super-Kamiokande.

The strongly favored explanation of solar neutrino flavor change is the Large Mixing Angle version of the MSW effect, with the best fit parameters $\Delta m_{12}^2 = 7.1 \times 10^{-5}\ \text{eV}^2$, $\sin^2 2\theta_{12} = 0.8$.



$P(\nu_e \to \nu_e)$
$4E_\nu/\Delta m^2\ (eV^{-1})$
vacuum
osc.
MSW matter
osc.
0.02
0.1
0.3
0.8

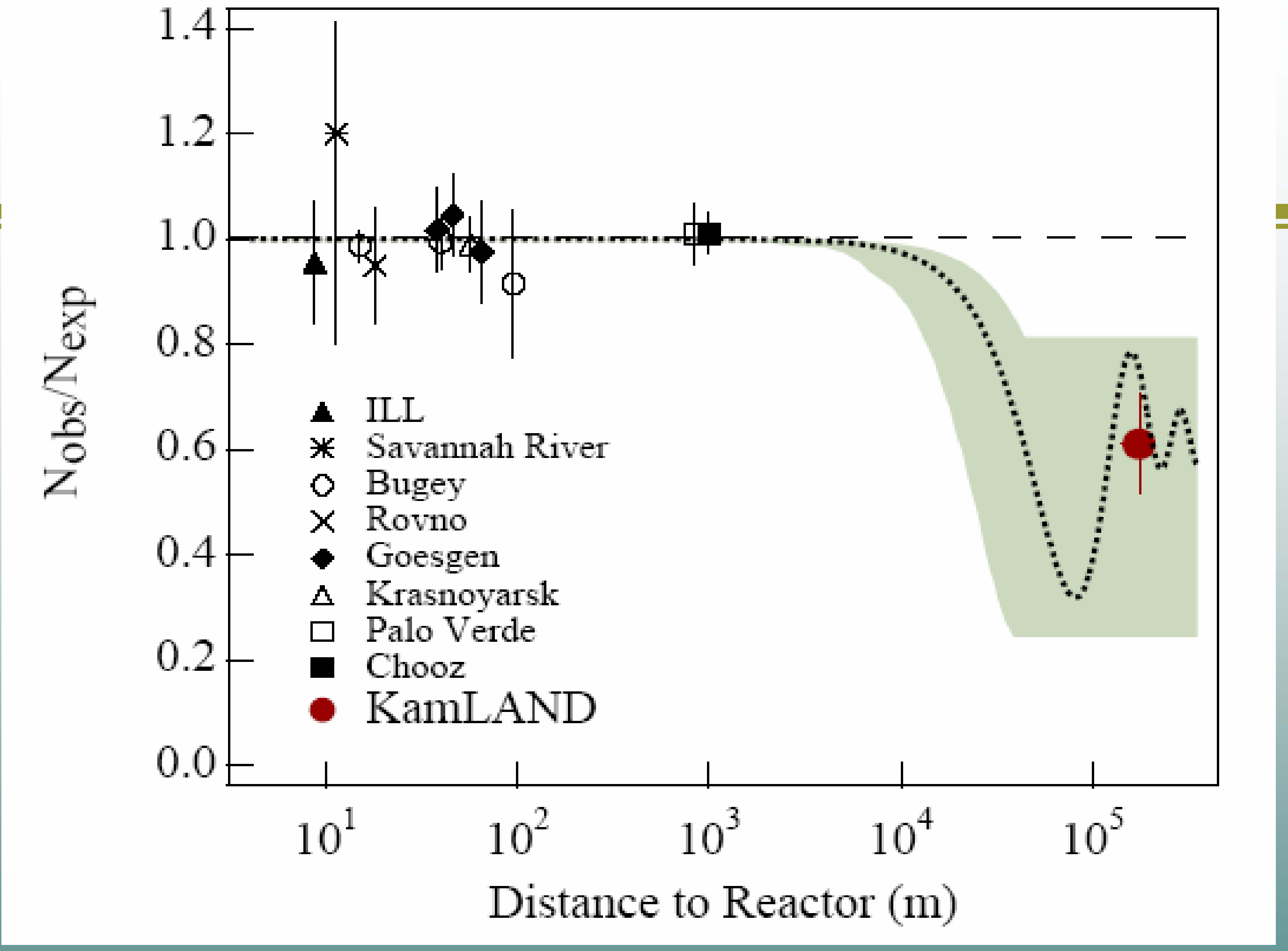
Disappearance experiments:

Reactors are source of $\bar{\nu}_e$ through the neutron β-decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

and experiment looks for a possible decrease in the $\bar{\nu}_e$ flux as a function of distance from the reactor, $\bar{\nu}_e \rightarrow X$ [if converted to $\bar{\nu}_\mu$, say, one would see nothing, $\bar{\nu}_\mu$ could have produced μ^+ but does not have sufficient energy to do so].

- Kamland experiment confirms that $\bar{\nu}_e$ do indeed disappear when the reactor $\bar{\nu}_e$ have travelled ≈ 200 km. $\bar{\nu}_e$ flux is only 0.611 ± 0.085 ± 0.041 of what it would be if none of it were disappearing. Interestingly this reactor $\bar{\nu}_e$ disappearance and the solar neutrino results can be describe by the same neutrino mass and mixing parameters



This gives confidence that the physics of both phenomenon has been correctly identified.

Neutrino Mass Matrix

As discussed the data from solar and atmospheric neutrino and reactor antineutrinos experiments provide evidence for neutrino mass and mixing with two different mass scale and large mixing angles.

$$\Delta m^2_{\mathrm{atm}} \equiv \Delta m^2_{23} = (2.0 \pm 0.5) \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2\theta_{23} \equiv \sin^2 2\theta_1 = 1.00 \pm 0.04$$

$$\Delta m^2_{\mathrm{solar}} \equiv \Delta m^2_{12} = (7.1 \pm 0.6) \times 10^{-5}\,\mathrm{eV}^2$$

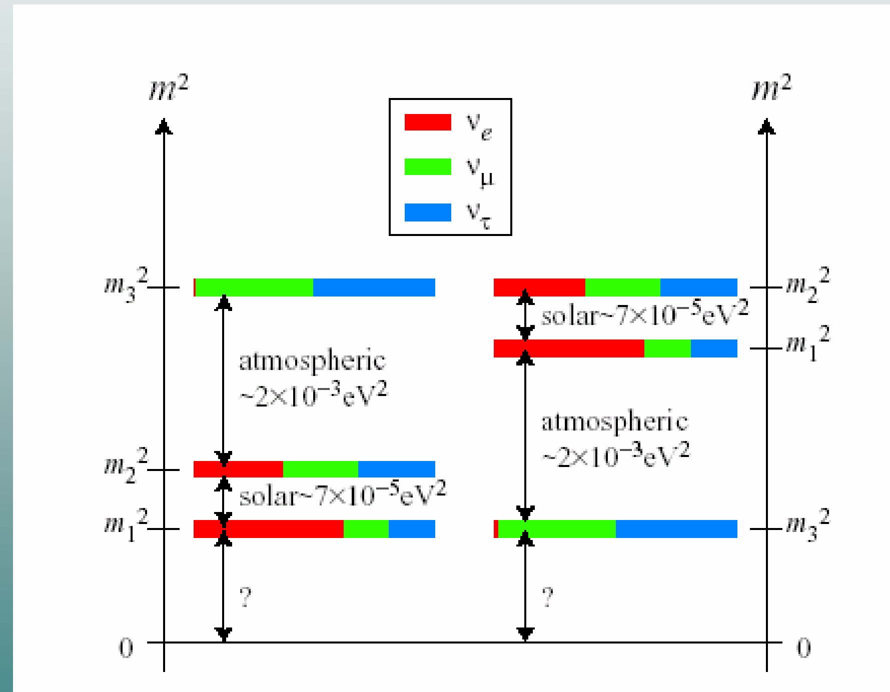
$$\tan^2\theta_{12} \equiv \tan^2\theta_3 = 0.45 \pm 0.06\,.$$

Further the CHOOZ experiment give

$|U_{e3}|^2 \equiv \sin^2\theta_2 < 4 \times 10^{-2}$

Since the oscillation data are only sensitive to mass squared differences, they allow for 3 possible arrangements of the different mass levels.

Degenerate neutrinos i.e. $m_1 \approx m_2 \approx m_3$

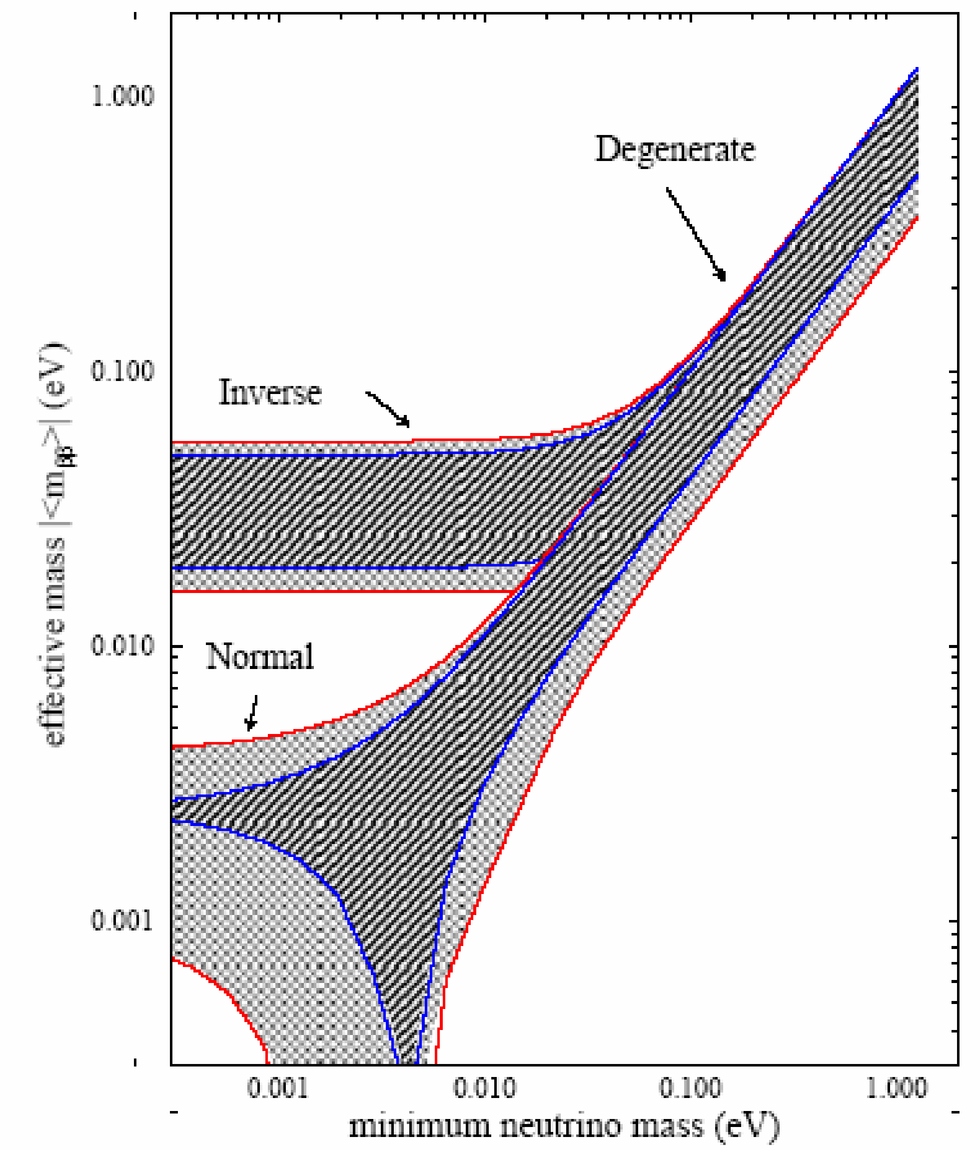


Riazuddin, JHEP 0310 (2003) 009

For the degenerate neutrino mass

m1 ~ m2 ~ m3 >> $\sqrt{\Delta m^2_{32}}$ = 0.045

the effective mass in neutrinoless double β-decay is larger than ~ 0.05 eV, constrained from above by the mass limit from tritium β-decay



1.000
0.100
0.010
0.001
effective mass $|\langle m_{\beta\beta}\rangle|$ (eV)
Degenerate
Inverse
Normal
0.001
0.010
0.100
1.000
minimum neutrino mass (eV)

If effective mass is confirmed to be

$$0.39^{+0.17}_{-0.28}\ \text{eV}$$

It would strongly indicate that neutrinos follow degenrate mass pattern, when

$$\Delta m^2 / m^2 \ll 1$$

Conclusion

- To conclude various neutrino mass patterns and corresponding neutrino mass matrix types are possible. Further the absolute value of neutrino mass is not yet determined. However, one thing is certain that neutrinos are providing an evidence for new physics but the scale of new physics is not yet pinned down. The heavy right handed neutrinos at new physics scale may provide an explanation for baryogenesis through leptogenesis. If past is of any guide, neutrinos will enrich physics still further.